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# A LABORATORY MANUAL
# OF
# EXPERIMENTS IN PHYSICS

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# A LABORATORY MANUAL
# OF
# EXPERIMENTS IN PHYSICS

BY

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FIRST EDITION

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# PREFACE

This manual is the outgrowth of the one used for many years in the University of Wisconsin, for which much credit is due to Dr. O. A. Gage, formerly in charge of the elementary laboratory at this university, as well as to his predecessors, particularly Professor L. F. Miller. Since the manual was prepared especially for the students at the University of Wisconsin, many of the directions are addressed particularly to them, with occasional local references, but this arrangement should not in any way lessen the usefulness of the volume to others.

As some of these students have had no previous training in physics, a number of experiments of rather elementary caliber are included, but, on the other hand, there are many which will prove all the average sophomore can manage. It is believed, too, that some of the experiments will be found a little out of the ordinary, while a few are probably unique—at least, in their adaptation to elementary work.

L. R. INGERSOLL.

MADISON, WIS
*March*, 1925.

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# A LABORATORY MANUAL OF EXPERIMENTS IN PHYSICS

## INTRODUCTION

### LABORATORY RULES

*Each student is asked to read carefully and abide by the following laboratory rules:*

**1. Assignments.**—After registration in the course, the student should report at his first regular laboratory period in the particular laboratory to which he has been assigned. His attendance will be checked here and an experiment given out. Notices regarding conflicts will be found on the bulletin boards, and these must be arranged at the earliest possible moment. As soon as possible after the opening of the term, each student will be assigned to an instructor, who will read his notebook and have direct supervision of his work. He will also be given a numbered pigeonhole in which will be kept his laboratory manual, notebook, and assignment card.

**2. Attendance.**—Each student is expected to attend regularly at his assigned periods and to be in the laboratory at the beginning of each period. Tardiness at the beginning of the period or early leaving at the end will be noted by the instructor and account taken of it in subsequent reports on attendance. As the records of attendance are kept on the assignment cards, the student is asked to *report regularly at the desk at the beginning of each period* to have his attendance checked.

Absences must be made up by arrangement with the instructor within 10 days of their occurrence. If a student has been ill and presents a doctor's excuse, this time may be extended somewhat. In general, however, the student will be expected to put in the full number of hours required for the semester's work—save a possible 10 per cent cut allowance—before he can be given credit for the course.

**3. Experiments.**—Experiments will be assigned from lists posted on the bulletin boards for the appropriate course. The minimum number of experiments which each student will be required to perform during the semester will also be posted. The average student is expected to exceed this number by one or more, and only in rare instances will more time than the usual number of hours a week be necessary for the completion of this list. It should be thoroughly understood that this is merely a *minimum* number of experiments and that the student is expected to work as many in excess of this as possible.

**4. Data.**—When an experiment has been assigned, the student should at once read over the accompanying description in the manual. Profitable use may also be made of reference books, which will be found in the laboratories. When he has mastered as thoroughly as possible the principles of the experiment—and not until then—the student is ready for the actual measurements, and these observations, usually called the data of the experiment, are to be recorded either in ink or in pencil *upon one of the left-hand pages of the notebook, or, in courses in which the loose-leaf system is used, on the yellow data sheet.* In general, this sheet should be ruled by the student so that the data may be taken down in tabulated form. This data sheet or record of original observations is, in a sense, the most important part of the entire experiment. It should contain *all* the measurements and must under no circumstances be altered or destroyed but should be carefully preserved and presented with the final report. If a mistake is made, cancel the wrong part with a line. *An erasure on the data sheet is never permissible and will, in general, cause the entire experiment to be rejected.*

When, as is customary, two students work together at an experiment, each should keep an independent and complete data record. If the recording can be done by only one student, such record is to be marked "Original" and the other "Copy." Each partner should take his turn at reading the instruments. Be very careful to state the units in which all measurements are made. *Record the date on which each set of observations is taken.*

While many of the measurements made in the physical laboratory are within the scope of everyday experience, *e.g.*, the use of a rule in measuring lengths, there are certain requirements in scientific work which may be new to the student but with which he must familiarize himself at the earliest opportunity. The first of these is the number of determinations to be made in any particular measurement. It is a fundamental law of laboratory work that a *single* measurement is of very little value because of the liability not only to gross mistakes but also to smaller errors. Accordingly, it is customary to repeat all measurements so that the total number of observations of a particular quantity is seldom less than three and in some cases even 10 or more. The *average* of these readings is, obviously, of a greater probable accuracy than any one could possibly be alone. The number of readings to be taken is usually specified in the earlier experiments, but it is expected that the student will soon accustom himself to this requirement and always take his measurements in sets of three or more, whether this is explicitly specified or not.

Requirements of accuracy demand that each measurement be made as carefully as possible. To fulfil this requirement, it is universal practice in physical measurements to estimate the reading of a scale to tenths of the smallest division. Thus, if a scale is divided into millimeters, as the ordinary meter stick, the reading will be expressed to tenths of a millimeter, *e.g.*, 4.3 mm., 27.42 cm. In case the reading falls exactly on a scale division, the tenths are expressed by 0, *e.g.*, 6.0 mm., 48.50 cm.

Too much insistence cannot be laid upon both of the above points, *viz.*, the repetition of observations and the reading to tenths of the smallest scale division. *No reports can be accepted which are deficient in either of these respects.*

**5. Computation.**—Computations may be made on loose sheets of paper if desired. It is expected that all engineers will provide themselves with slide rules, and all other students are urgently requested to accustom themselves to the use of logarithms in computing. This request applies whether or not the student is already familiar with logarithms. Simple logarithm tables will be found at the end of this manual and half an hour's application to the instructions preceding them should render it possible for any student to use them with reasonable facility.

Too much insistence cannot be placed upon this point, for the use of either logarithms or the slide rule will save many hours of tedious arithmetical computation.

**6. Reports.**—After the data have been taken and computations made, they should be approved by the instructor. The final report is then to be written *in ink* on the *right-hand* pages of the notebook, or, in the loose-leaf system, on the white sheets. This report should embody:

1. Name and number of experiment. Name and number of student and partner.
2. Object of the experiment (to be stated in the student's own words).
3. Apparatus used, with diagram; give numbers of apparatus when possible.
4. Description of how the experiment was performed.
5. Method of deducing results from original data.
6. Summary of data, and results; be sure to specify units, and, where possible, place side by side theoretical values and actual results, for purposes of comparison.
7. Curves, if required.
8. Physical interpretation of results and answers to questions.

It is also frequently possible and profitable to include a discussion of sources of error and their elimination.

Such reports are, in general, to be written during the regular laboratory periods. In exceptional cases, however, the student, *with the permission of the instructor*, may take his book home for outside work.

When an experiment has been completed and the report written, the student should at once consult the instructor, who is to talk over the report very briefly with him, clear up any difficulties which may be outstanding, and assign a new experiment. Later the instructor will make a more careful survey of the report, at which time he will stamp the date on which it is read. If the report is satisfactory, it will be also stamped "Entered" and given a grade. If merely the date has been stamped, it means that the report needs correction. Such corrections must be made immediately, *i.e.*, before further work is performed, and can, in general, be made without materially defacing the report. If, however, the difficulty is fundamental, it may be necessary to rewrite the report completely, in which case the student should turn to a fresh sheet and begin over again. All data and reports, whether satisfactory or otherwise, must be saved. *Under no circumstances may any sheets be cut or otherwise removed from the book.*

The answers to the questions on each experiment should constitute one of the most important parts of the report. These must in every case be written—preferably at the end of the report. One of the commonest questions is "Interpret the curve." This means to state what physical law or conclusions may be drawn from such a curve. For instance, if a curve of distances covered by a falling body as abscissas, and squares of the time as ordinates, should come out as a straight line passing through the origin, the conclusion would be that the space covered by the falling body is directly proportional to the square of the time. Needless to say, the "optional questions" are for the ambitious student who wishes to complete his work in the best possible manner.

When a report is completed, enter the number of the experiment with the date in the first (inside) cover of the book, which may be ruled for this purpose. In general, the student must finish each experiment, *i.e.*, report, before another is started, but occasional exceptions, for good reason, may be made to this rule. Leave a single blank page at the end of each report.

**7. Special Instructions for Courses in Which the Loose-leaf System Is Used.**—All *data* sheets must be *in triplicate* (unless partner is absent). Use clip board, two carbons, and *yellow* paper (size 7¼ by 10) for taking data. When data taking is completed, have sheets initialed by instructor, who will then retain one copy while each of the other two copies is incorporated in the completed reports. *No changes of any sort may be made on data sheets after they have been initialed and filed.* Reports are to embody (1) the yellow data sheet, (2) the report written according to the foregoing instructions and embodying a brief resumé of the data, and (3) curves. The reports are to be written on the *white* paper (same size as yellow).

**8. Significant Figures.**—Students unacquainted with laboratory work in physics waste considerable time in needless computations; they do not recognize the fact that, since physical apparatus is far from perfect, the results obtained from it cannot be absolutely correct. No matter, therefore, to how many places the computations may be carried, the accuracy of the result cannot exceed the accuracy of the data. If an instrument is used twice to measure the same thing, the results will not be identical. For example, if a certain distance be measured by a common meter stick, the results may be 48.25 cm., 48.23 cm., 48.22 cm., the reason for the variation being that the finest divisions on the meter stick are 0.1 cm. and hence the fourth figure is obtained by estimating tenths of that division. The average of the three readings is 48.233333 cm., but, as the distance is not known accurately to a hundredth of a centimeter, it is unnecessary to carry the result beyond this place. Therefore, the average should be given as 48.23 cm. Mathematically, the numbers neglected have a meaning, but physically they do not, and may be omitted, for only the first three figures are reliable and the fourth one is in doubt. Hence, by the term "significant figures" is meant those figures in a result which are trustworthy and have some significance. Obviously, the figures after the third decimal place in the length just mentioned are of no value, and so are not significant figures. The position of the decimal point in no way affects the number of these figures; this number is determined entirely by the accuracy of data. Suppose three significant figures are to be retained in the following numbers: 1763298.23 and 0.0003628. Then they should be written 1760000 and 0.000363.

In work of the character done in this laboratory it is usually unnecessary to have more than four significant figures in the result, but the following rules will be of help in determining how many figures to retain:

1. In addition and subtraction, do not carry the result beyond the first column which contains a doubtful figure.
2. In multiplication and division, the number of significant figures in the result should be one greater than the smallest number of trustworthy figures contained in any factor used in obtaining the result.

These rules give the number of significant figures which should appear in the result, the last figure being always in doubt; but, in computing, it is better to carry one more figure than they specify. The following examples illustrate the principles just mentioned:

$$4567 + 1.48 + 0.0764 = 4568.6$$
$$13.28 \times 2.06 = 27.36$$
$$0.0735 \times 0.002 = 0.00015$$
$$189324500 \times 66 = 12500000000 = 125 \times 10^8.$$

As already mentioned, a slide rule gives results which are sufficiently accurate for most of the experiments and saves much time. The course for engineers has been planned with the assumption that all engineers own and use them, and the students in this course will work at a disadvantage without them.

**9. Plotting of Curves.**—In the plotting and discussion of curves, the following terms are frequently used:

The *abscissa* is the distance $OA$ (Fig. 1) measured along the horizontal line $OX$. This line is called the axis of abscissas or $X$-axis.

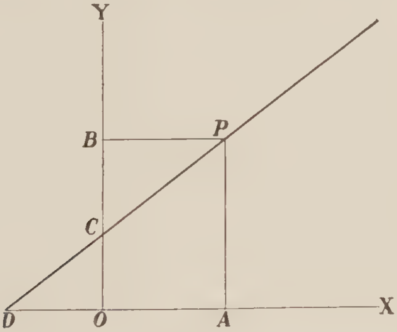


Fig. 1.

The *ordinate* is the distance $OB$, measured along the vertical line $OY$. This line is spoken of as the axis of ordinates or the $Y$-axis.

*Coordinates* of the point $P$ are the two distances $OA$ and $OB$.

The *origin* is the point $O$, the intersection of the two axes. This point is called the origin only when the magnitudes plotted on the two axes have their zero values at this point.

An *intercept* is the distance measured from the origin along one of the axes to the point at which the curve meets the axis. Thus, $OC$ is the $Y$-intercept and $OD$ the $X$-intercept for the curve in Fig. 1.

The slope of a curve is a measure of the angle which the curve makes with the $X$-axis. (It is really the trigonometrical tangent of this angle.) Thus, for the curve in Fig. 1, the slope determines the angle $PDA$. If one curve has a greater slope than a second, it means that the angle formed by the first curve with the $X$-axis will be larger than that formed by the second.

Plotting paper of the same size as the pages in the notebook is to be purchased and always used. Certain conventions observed in plotting curves have been included in the following rules and should be observed:

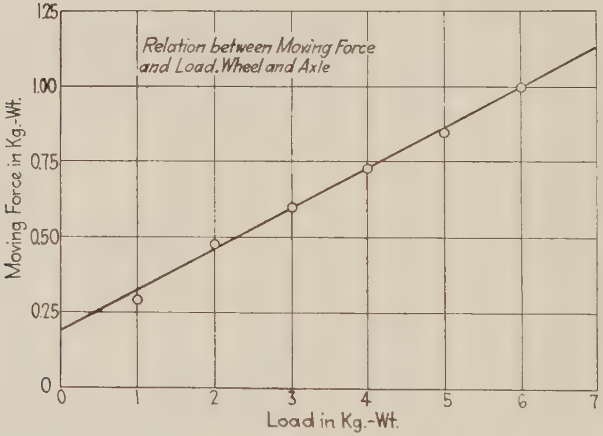


FIG. 2.

1. Choose such scales that the curve will cover as nearly as possible the whole sheet, but make them convenient, *i.e.*, make each small division equal to one, two, four, five, or ten units. The same scale need not be used on both axes.

2. Indicate the scale on the axis together with the name of the quantity represented. The numbers should increase from left to right and from the bottom of the sheet upward.

3. Represent the points corresponding to the experimental values by crosses, or dots enclosed in circles.

4. Draw as smooth a curve as possible through the points. Do not make the curve irregular in order to allow it to pass through all the points.

The above rules are illustrated in the following example:

| Load, Kilograms | Moving Force, Grams |
|---|---|
| 1 | 300 |
| 2 | 475 |
| 3 | 600 |
| 4 | 725 |
| 5 | 850 |
| 6 | 1000 |

**10. Methods of Averaging.**—When several values of a physical quantity, as the specific heat of a substance or the focal length of a lens, have been determined, they usually differ among themselves, and the most probable value is the arithmetic mean. If it is necessary, however, to find the average difference in certain series of readings (*e.g.*, to find the average length of the bricks in a wall by noting on a tape the reading at each junction of two bricks), the arithmetic mean of the various differences should not be used, because only the first and the last readings affect the result, and all intermediate values have no influence. Accordingly, a special form of computation must be used.

As an illustration of this method, the ratio of the inch to the centimeter will be obtained from the following data:

| No. observations | Readings | | Difference in successive readings in centimeters | Differences between readings | | |
|---|---|---|---|---|---|---|
| | Inches | Centimeters | | | Inches | Centimeters |
| 1 | 8.50 | 18.60 | 5.07 | Fifth—First | 8.00 | 20.30 |
| 2 | 10.50 | 23.67 | 5.06 | Sixth—Second | 8.00 | 20.32 |
| 3 | 12.50 | 28.73 | 5.09 | Seventh—Third | 8.00 | 20.35 |
| 4 | 14.50 | 33.82 | 5.08 | Eighth—Fourth | 8.00 | 20.33 |
| 5 | 16.50 | 38.90 | 5.09 | | | |
| 6 | 18.50 | 43.99 | 5.09 | | 32.00 | 81.30 |
| 7 | 20.50 | 49.08 | 5.07 | | | |
| 8 | 22.50 | 54.15 | | | | |

By averaging the successive differences between the centimeter readings, 5.078 is obtained and, as this corresponds to 2 in., the ratio of the inch to the centimeter is found to be 2.539. If the readings of observation 1 are subtracted from the corresponding ones of observation 8, it is found that 14 in. correspond to 35.55 cm., or the ratio of the inch to the centimeter from these readings is equal to 2.539. This shows that the value obtained from the average differences between successive readings is the same as that found by using only the first and last readings in columns 2 and 3. In order to have the intermediate readings influence the results, the method of computation indicated in the last three columns may be used. By adding these columns it is found that 32 in. correspond to 81.30 cm., or 1 in. equals 2.5406 cm. The true value is 2.5400.

Another useful application of this method of averaging is in the determination of the periodic time of a vibrating body. By this term is meant the time which elapses between two successive passages of the body through a point, in the same direction. A convenient form for recording the data is shown in the following table:

| Number | Vibrations | Time | | | Difference | |
|---|---|---|---|---|---|---|
| | | Hours | Minutes | Seconds | Minutes | Seconds |
| 1 | 0 | 2 | 35 | 50 | | |
| 2 | 100 | .. | 39 | 9 | 3 | 19 |
| 3 | 200 | .. | 42 | 29 | 3 | 20 |
| 4 | 300 | .. | 45 | 48 | 3 | 19 |
| 5 | 400 | .. | 49 | 9 | 3 | 21 |
| 6 | 500 | .. | 52 | 30 | 3 | 21 |
| 7 | 600 | .. | 55 | 50 | 3 | 20 |
| 8 | 700 | .. | 59 | 10 | 3 | 20 |

If the average of the differences given in the last column is taken, it shows that the periodic time is just 2 sec., since 100 vibrations occurred in 200 sec. When the first and last readings are used the same value results. As was shown in the preceding illustration, there is a simple way of including these intermediate readings as indicated in the accompanying table:

| | Vibrations | Difference | |
|---|---|---|---|
| | | Minutes | Seconds |
| Fifth—first | 400 | 13 | 19 |
| Sixth—second | 400 | 13 | 21 |
| Seventh—third | 400 | 13 | 21 |
| Eighth—fourth | 400 | 13 | 22 |
| | 1600 | 53 | 23 |

Therefore one vibration takes place in 2.002 sec.

This seems but a slight difference, but in cases where the differences show greater disagreement, the variation is more pronounced, and the method just described is capable of giving surprisingly accurate results. *It must be kept clearly in mind, however, that this method is to be applied only when the average difference of a series of readings is desired.*

**11. Errors.**—*Absolute accuracy* is, of course, unattainable in laboratory measurements. Every result, no matter how carefully obtained, has a certain "probable error," which depends on the number of measurements made, their concordance, and some other factors. It should be the aim of the student to make his measurements with the greatest accuracy attainable with the given apparatus; in no case, however—except by accident—will his results agree exactly with the true values of the quantities measured. Thus, if the accepted value of the acceleration of gravity at a certain place is 981.4 cm. per second per second and if the probable error of determination for the apparatus used by the student is of the order of 5 cm. per second per second, a determination of 981.3, for example, can be considered only as a result of pure chance and no more worthy of credit than one of, say, 976.8.

The accuracy with which the different measurements in a particular experiment are to be determined depends upon the way these quantities are to be used in the computations. Unless the student possesses some knowledge of these relations, time may be wasted in making a measurement with too little or with too great care. In the discussion which follows frequent reference will be made to the actual error and the relative, or percentage, error. The actual error is the amount by which the experimental value varies from the true one; the relative error is the ratio between the actual error and the true value, and when this ratio is expressed as a percentage it is then called the percentage error. For example, if a 20-gm. body is weighed on a balance, its mass may be found to be 20.4 gm. The actual error is 0.4 gm.; the relative error is 0.02; the percentage error is 2 per cent.

If quantities are to be added or subtracted, it is the actual error which is of importance. Thus, if the distances to be added together are 2500 cm. and 1.38 cm., it is obviously absurd to include the two decimals unless the greater length has been measured to an accuracy of at least 0.1 cm. From another point of view, if the greater distance has been measured roughly and an error of 10 cm. has been made, the sum obtained by adding the two quantities means little. In multiplication, quite a different state of affairs exists. As an illustration, the area of a rectangle the sides of which are 2000 cm. and 1 cm. is to be found. One observer makes a mistake of 10 cm. in the base but determines the altitude correctly. This gives an apparent area of 2010 sq. cm. If a second observer measures the base correctly but makes a mistake of 0 01 cm. in the altitude, then the area will appear to be equal to 2000 × 1.01, or 2020 sq. cm. Thus, an absolute error of 0.01 cm. in one case has a greater effect than one of 10 cm. in the other. The relative error, however, in the second measurement is 0.01, and in the first 0.005. This is an illustration of the general fact that in multiplication and division the relative, and not the actual, error is all-important in affecting the result.

This relationship frequently makes the experimental work much easier. Thus, for the area just mentioned, the base need not be measured by an instrument as accurate as that needed for the altitude. Even if the altitude is determined by micrometer calipers capable of measuring to 0.001 cm., an ordinary meter stick will measure the base with a smaller percentage error. Accordingly, it is unnecessary to use a more accurate instrument in determining this length.

If a particular quantity is raised to some power and then multiplied or divided by some other quantities, its effect upon the result is equal to its relative error multiplied by the power to which it is raised. If in the same kind of an operation a square root is taken instead of a square, its effect is equal to one-half of its relative error. Accordingly, quantities which are raised to some power must be determined with a greater relative accuracy than other quantities. Numerous illustrations of these principles will be found in the experimental work outlined in this manual.

**12. Synopsis of Laboratory Rules.**—

1. Be punctual; habitual tardiness will be counted as absence.

2. Absences must be made up. If possible, this should be done under the student's own instructor. In any case, the student should be sure that the instructor enters the credit on his work card.

3. Follow the laboratory bulletin board.

4. Do not start an experiment until it has been regularly assigned and the assignment noted on the work card.

5. Under no circumstances may a partner who was absent when a particular experiment was performed use the data taken when he was away.

6. Students are not to move apparatus about the room without permission, or to go behind the desk. Special cooperation is asked in keeping apparatus and laboratory in as good shape as possible.

7. The laboratory will, in general, close for the semester on the Saturday noon before examinations begin. Not more than two experiments will be accepted during the last week.

8. Occasional oral or written quizzes will be given on the laboratory work and in the last period of the semester a final laboratory examination will be held. On application to instructor, students will be allowed to take notebooks home in preparation for this examination.

# PART I

# PRELIMINARY MEASUREMENTS

## EXPERIMENT 1

### The Rule

Object: *To become familiar with some of the principles of physical measurement through a simple experiment.*

A ruler and a rectangular block of wood will be furnished; measure the dimensions of the latter carefully and compute its volume. This is a very simple experiment but one which can be made to illustrate some of the principles of laboratory practice already outlined in the Introduction. Read the Introduction through carefully, paying particular attention to Pars. 4 to 9.

Some of these principles may be comprised in a simple set of rules as follows:

1. *Make several measurements* of each dimension and take the mean or average. In the present case a suitable number would be five for each dimension, made on different parts of the block. The average of these will be obviously of greater accuracy than any single measurement. In making these measurements it is not generally desirable, nor, in fact, always possible, to use the end of the rule as the zero from which to start the measurement. It is good practice to place the rule on the block so that one edge of the latter comes at some scale division and subtract this reading from that at the other end.

2. *Always estimate to tenths of the smallest scale division.* In this case the metric scale divided into millimeters will be used. The readings then should be *to tenths of a millimeter.* Expressing each dimension in centimeters, sample readings might be 6.45 cm., 9.72 cm., etc. In case the reading falls exactly on a millimeter division, *express the tenths by a zero, e.g.,* 4.50 cm., and in

the rare case of its falling exactly on a centimeter division write it 6.00 cm., 3.00 cm., etc. This shows that the worker has attempted to make his readings accurate to the tenth part of the millimeter in each case. Skill in estimating tenths is easily and rapidly acquired and is of the highest importance in all physical laboratory work.

3. *Do not retain too many significant figures in the result.* As an illustration, suppose the mean values of the dimensions of the block are: 6.27 cm. wide, 8.31 cm. long, and 2.45 cm. thick. Multiplying these, the result is, for the area of a side of the block, 20.3595 sq. cm., and for the volume, 127.654065 cc. Now it is evident that the accuracy of the measurements will not justify retaining more than four, or at most five, significant figures in the computed results; so considerable labor will be saved in calculation by dropping off these unnecessary figures, calling the area of one side 20.36 sq. cm. and the volume 127.65 cc. In this connection read the discussion of Significant Figures in Par. 8 of the Introduction.

Tabulate the results in three columns, one for each dimension. In calculating the volume, use the average of the results for each dimension.

**Errors.**—First read the discussion of this subject in Par. 11. Now, assuming that an error of 0.1 mm. is made in the measurement of each dimension, calculate what this will be in terms of percentage error in each case.

If the student were to make a *very great many measurements* of the foregoing quantities, certain facts would appear which cannot be made clear when the determinations are limited in number. Inasmuch as the 100 or more measurements required to bring out these facts would take an unnecessary amount of time, the following data are given on an experiment of this sort and will be discussed: The student was asked to make 230 measurements of a certain length with an ordinary meter stick. His measurements when tabulated were as follows:

| Measurements, Centimeters | Number of Each |
|---|---|
| 77.30 | 2 |
| 77.31 | 3 |
| 77.32 | 7 |
| 77.33 | 8 |
| 77.34 | 16 |
| 77.35 | 25 |
| 77.36 | 35 |
| 77.37 | 36 |
| 77.38 | 33 |
| 77.39 | 24 |
| 77.40 | 19 |
| 77.41 | 10 |
| 77.42 | 7 |
| 77.43 | 4 |
| 77.44 | 1 |

The correct value of the length was 77.37 cm. and it will be noted that there are more measurements of about this value than for any corresponding value. In other words, there are a great many small errors and relatively few large ones. Now if a curve is plotted with the error for each set of measurements, *i.e.*, 0.1, 0.2 mm., and so on, as abscissas and the number of readings for each class of measurements as ordinates, a symmetrical curve will result which is known as the "error curve."

**Curve.**—Plot the above list of measurements, using as abscissas the values of the errors, considering 77.37 cm. the correct length, and the number of such errors in each class as ordinates. Put the origin in the center of the lower edge of the paper and follow the instructions given in Par. 9 of the Introduction.

### Questions

**1.** What is the volume of the block in cubic centimeters, cubic millimeters, and cubic decimeters? What is its density? (Weigh block on a trip balance and divide mass by volume. See first paragraph of Exp. 3.)

**2.** To secure the same percentage error in the three measurements which dimension must be measured the most carefully and why?

**3.** From the above error curve what can be said as to the relative frequency of occurrence of large and small errors?

**4.** Is there any relation between the above curve and the error curve formed by the probability (shot) board in the Physical Museum?

## EXPERIMENT 2

## The Vernier

OBJECT: *To become acquainted with the principle and the use of the vernier.*

One of the commonest attachments to a measuring instrument is a vernier, which enables a fraction of the finest division on the main scale to be determined accurately. In the present experiment the theory of the vernier will be studied and the method of using it mastered.

The vernier is an auxiliary scale, graduations of which are different from those of the main one, but bear a simple relation to it. The vernier represented in Fig. 3*a* is so arranged that it reads to fifths of a division. That such a reading may be made, five divisions on the vernier must equal four on the scale; accordingly, each division on the vernier is one-fifth of *one scale division shorter* than a scale unit. This difference in length of the two divisions is called the "least count" of the vernier. An inspection of the figure will show why a reading to fifths of a unit can be made. The third unit on the vernier coincides with the fifth on the scale; hence, division 2 is 0.2 units from 4; division 1, 0.4 units from 3; and the vernier zero, 0.6 units from 2. Consequently, the reading is 2.6 units. A very common vernier is one in which 10 vernier divisions equal 9 scale divisions. In this case, then, each vernier division is nine-tenths of a scale division, *i.e.*, it falls short by one-tenth, and the least count is, accordingly, one-tenth of a main scale division.

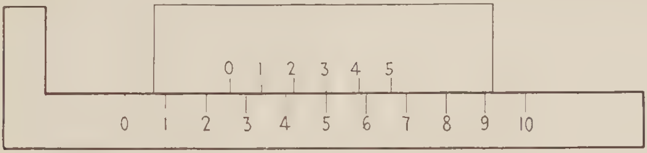


FIG. 3*a*.

Hence, in using a vernier, two readings must be taken, one on the scale and one on the vernier. The scale reading is determined by the zero of the vernier. This zero either coincides with some one scale division or lies between two of them. In the first case the scale reading is the number of the division which is in coincidence; in the other case it is the number corresponding to the smaller of the two divisions. The vernier reading is the number of its division which coincides with some one scale division. To obtain the actual reading, these two must be combined by adding to the reading determined by the zero of the vernier the product of the vernier reading times the "least count" of the instrument.

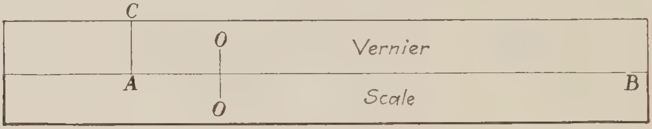


Fig. 3*b*.

The first step in this experiment is the construction of a scale and vernier. A piece of cardboard will be furnished. Draw on it the lines *AB* and *AC*, as indicated in Fig. 3*b*. About a centimeter to the right of *A* and below the line *AB* draw carefully the scale which is specified by the instructor. Then above the line construct the required vernier, making sure that the two zeros coincide. In dividing the vernier it will be found worth while

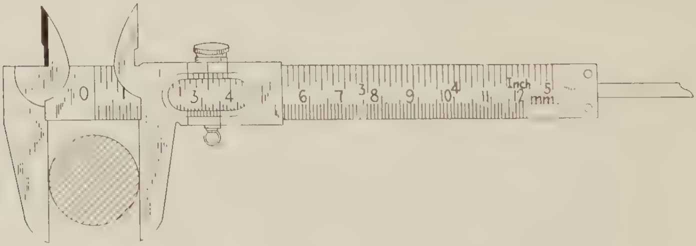


Fig. 4.

to make use of a well-known construction of elementary geometry. Draw two lines from a single point, making an angle. With a rule mark off carefully on one the total length of the vernier, measured from the intersection. On the other mark off a series of equal divisions (measured with the rule) the same in number as the parts into which the vernier must be divided. Connect the end of the vernier with the last division and draw lines parallel to this through the other divisions. This will give a fairly accurately divided vernier. Now cut along the lines *AB* and *AC*, and a crude vernier caliper will have been constructed of the same form as shown in Fig. 3*a*. Measure with it the length of a cylinder. Redetermine the length of the same cylinder, using a steel vernier caliper. The cardboard caliper is to be pasted on the data sheet of this experiment, with jaws open and the reading indicated.

Finally, measure some object which is about as large as can be taken by the steel calipers, using both the inch and the centimeter scales. Compute from this result the ratio of the inch and the centimeter.

Tabulate all the data obtained as follows:

| Object measured | Kind of vernier caliper | Length of scale division | Length of vernier division | Least count | Measurements |
|---|---|---|---|---|---|
| | | | | | |

### Questions

**1.** Explain in detail the vernier, including a description of each vernier used, stating the "least count" in each case.

### Problems for Experiment 2

| Problem | Length | Finest Scale Division | Vernier |
|---|---|---|---|
| 1 | 6 in. | 0.2 in. | To read to $1/40$ in. |
| 2 | 15 cm. | 0.5 cm. | To read to 0.5 mm. |
| 3 | 6½ in. | 0.25 in. | To read to $1/40$ in. |
| 4 | 6 in. | 0.5 in. | To read to $1/8$ in. |
| 5 | 15 cm. | 0.5 cm. | To read to 1 mm. |
| 6 | 6 in. | 0.5 in. | To read to $1/16$ in. |
| 7 | 6½ in. | 0.5 in. | To read to $1/10$ in. |
| 8 | 6½ in. | 0.5 in. | To read to 0.05 in. |
| 9 | 15 cm. | 1.0 cm. | To read to 2 mm. |
| 10 | 6 in. | 0.25 in. | To read to $1/32$ in. |

## Descriptive Note

**The Micrometer Caliper.**—The micrometer caliper is most convenient for the accurate measurement of short lengths. It consists of a carefully made screw, which is mounted in a strong frame *F* (Fig. 5). The object to be measured is placed between the end *B* of the screw and the anvil *A*. The distance through which the screw travels is measured by the two scales *S* and *T*. *D* is a device by which the same pressure is exerted by the screw each time; its use not only secures better results but also saves the micrometer screw from injury due to carelessly turning it down too tight. Is the reading affected by the rapidity with which the screw is turned?

In order to use the micrometer caliper, it is necessary to know the units in which the scale *S* is divided, and then to determine what fraction of this unit each division on *T* represents. In the micrometers used in this laboratory the scale *S* is divided into half millimeters. To find the value of the other divisions, turn the screw until the edge of the thimble, on which the scale *T* is marked, coincides with one of the divisions on *S*.

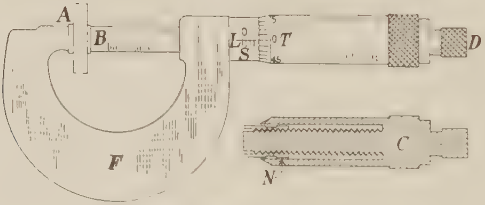


Fig. 5.

Then move the screw until this edge coincides with the next graduation. During this operation determine the number of divisions of *T* which pass the line *L*, and from these readings the value of one of these divisions may be readily found. The difference between the readings on the scales when the screw is in contact with *A* and when it holds the object against the anvil gives the dimension desired.

## EXPERIMENT 3

## Density of a Solid

OBJECT: *To determine the density of a solid in the form of a small cylinder; to acquire experience in the use of the micrometer caliper and the analytical balance.*

First read the preceding descriptive note.

By "density" is meant the mass of a unit volume of the substance; the obvious way, accordingly, of finding it is to determine the mass of a known volume of the substance and to divide this mass by the volume. The volume of any solid which has a simple geometric form may be readily computed from its dimensions. These are most conveniently measured by a vernier caliper or a micrometer caliper. First test the zero of the instrument by bringing the jaws together, always using the friction head *D*. Do this five times to see if the same pressure is used each time. Record these readings and take the average. This number is known as the "zero error" of the instrument. Then measure each dimension 10 times and take the average, correcting for the zero error. Do not take all the readings in one place, but make each determination at a different position; record all these readings on the data sheet.

To determine the mass of the substance, a sensitive balance is to be used. As this is a delicate piece of apparatus and must be handled with care, *the directions for its use given below should be followed exactly.* When the volume and the mass of the object have thus been determined, its density can be computed.

### Questions

**1.** Why should a number of readings of each dimension be taken? Give two reasons.

**2.** Which dimension of the cylinder should be measured most carefully and why?

### Descriptive Note

**The Analytical Balance.**—In the determination of mass, a sensitive balance is to be used, and, as it is easily put out of adjustment, it must be handled with great care. As shown in Fig. 6, the balance consists of a beam $A$, supported on a knife edge, to which two pans $P$ are attached at equal distances from the knife edge. These pans are themselves supported by secondary knife edges. When the pans are empty or contain equal masses, the beam comes to rest at its point of equilibrium, which is indicated by a long pointer moving over a short scale $S$. As the sensitiveness of the balance depends to a great extent upon the sharpness of the knife edges, these are protected, when the balance is not in use, by what is called an arrestment. This device lifts the beam and the pans off the knife edges and prevents them from moving; it is raised and lowered by a large knurled knob $B$ placed just beneath the floor of the case.

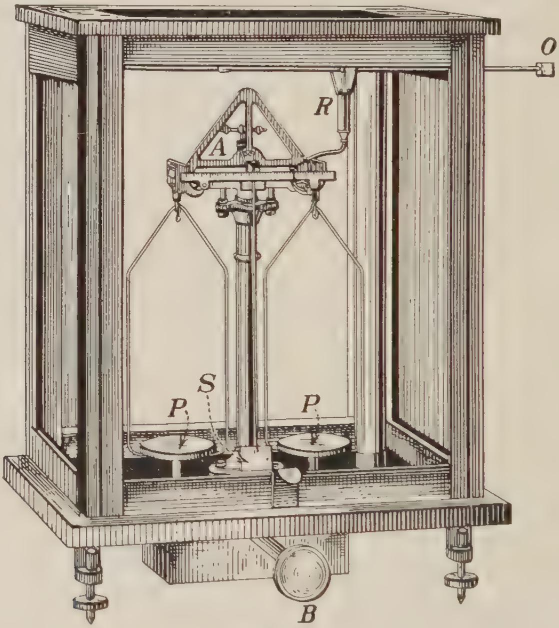


Fig. 6.

To find the mass of any body, it is placed in one pan (usually the left) and known masses—commonly (mis-) called "weights"—put in the other until the pointer indicates equilibrium. The term "weight" is used here in the commonly accepted sense, meaning one of the known masses required to counterbalance the mass of the body; the scientific distinction between mass and weight will be brought out later. As the balance vibrates for a long time, it is impracticable to wait for it to come to rest; so for ordinary work it is said to be in equilibrium when the pointer vibrates equal distances on each side of the rest point. This rest point, or the position of the pointer when the balance ceases to vibrate, is determined by releasing the balance with empty pans and allowing the pointer to swing through six or eight scale divisions. After the motion has become regular, determine two successive turning points of the pointer on the scale *S*, and the rest point is assumed to be midway between these readings. In making an actual determination of mass, always add sufficient weights to cause the pointer to vibrate equal distances from the true rest point and not from the central line of the scale, unless the balance happens to be so adjusted that this line is the true rest point.

Weights of less than 0.01 gm. are not put in the pan, but a rider is used. This is a wire stirrup weighing 0.01 gm. which can be placed in any position on the beam by an attachment *R*, this attachment being operated from outside the case by the rod *O*. The beam is graduated into 10 large divisions If the rider is on the tenth division, it acts as though placed in the pan. If placed on the fifth division, it corresponds to 0.005 gm. in the pan and similarly for the other divisions.

**Precautions:**

1. Before opening the case insert the knob and carefully lower the beam into position. If it vibrates freely, find the rest point. If it does not vibrate freely, call an instructor. The student should not attempt to adjust it himself.

2. Open the case and place the object on the left-hand pan and the weights on the right-hand pan until the addition of 0.01 gm. causes the indicator to move from the right of the rest point to the left. Then remove the 0.01 gm. and close the case. The final adjustment can now be made with the rider. *While adding or taking off weights or adjusting the rider*, be sure that the arrestment is up, for if the balance is free to swing while this is being done, it is usually thrown out of adjustment and injured.

3. The instrument is balanced when it moves through equal divisions on each side of the true rest point, but the total swing should not be more than six or eight divisions. For most work in this laboratory, it is unnecessary to determine the mass more closely than a milligram, since, in most cases, the tenths of milligrams are without meaning unless corrections are made which are usually omitted.

4. The special weights to be used with this balance are known as analytical weights. They are to be used only in connection with this balance and not for any other purpose. Always handle them with the forceps—never with the fingers; this rule applies to all the weights in the box, even the 100-gm. weights. **Each weight has a particular position in the box; be sure to return it to its proper place.** When the balance has been adjusted to equilibrium, be certain that the weights are added correctly. On the data sheet record the rest point taken at the beginning, the sum of the various weights used, and finally the rest point determined after the body has been weighed.

5. For accurate work a procedure somewhat more refined than the above is desirable. Determine the rest point $R_0$ with the pans empty, by averaging the mean of three successive turning points on one side with the mean of the two intervening turning points on the other. The rest point $R_1$ is then found in the same way during the process of weighing, when nearly in balance. Now, instead of trying to adjust the weights until $R_1$ exactly coincides with $R_0$, it is better to determine the *sensitivity* of the balance, or the number of scale divisions the rest point is shifted by the addition of 1 mgm. Having found this—by adding two or more milligrams if necessary—a simple interpolation will allow calculation of the change of weights which would bring $R_1$ into exact coincidence with $R_0$.

## EXPERIMENT 4

## Optical Lever

OBJECT: *To calibrate a screw and determine its pitch (linear displacement per revolution) by the optical lever.*

The optical lever is a device by which very small displacements may easily be determined. As used in this laboratory, it consists of a strip of brass supported by three needle points with a mirror attached at one end, and so mounted that it can be placed perpendicular to the strip. If a telescope and vertical scale are set up in front of the lever, any rotation of the latter about a horizontal axis may easily be detected. Figure 7 shows diagrammatically the arrangement of the apparatus. $S$ represents the screw, the pitch of which is to be determined, $B$ the indicator for showing the number of complete revolutions, $L$ the lever with the attached mirror $M$, $T$ the telescope. When the lever is moved through the angle $\alpha$, the direction from which the light which is reflected into the telescope comes suffers a change in angle $G$ ($= 2\alpha$) corresponding to the observed linear displacement $d$. As the distances are large and the displacement of the screw is comparatively small, certain approximations can be made. For example, the assumption in this experiment that the distance subtended on the scale by $\frac{1}{2}G$ is equal to $\frac{1}{2}d$, although not strictly true

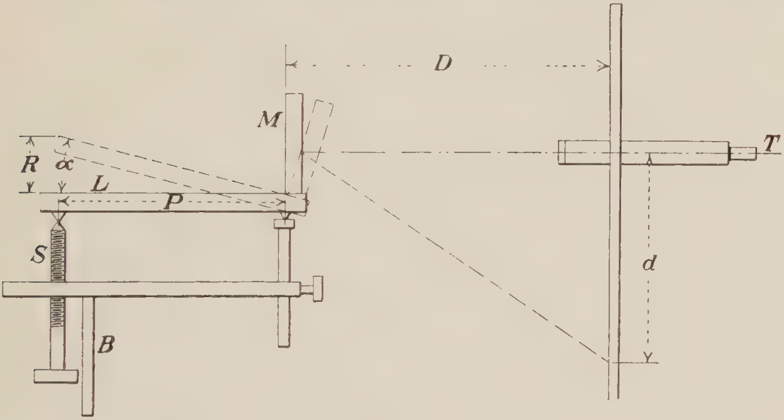


FIG. 7.

mathematically, would introduce less error than is produced by the inaccuracies of measurement. By admitting such an approximation, it can readily be shown by plane geometry that

$$R = \frac{Pd}{2D}.$$

Here, $d$ need not represent the distance from the zero to the reading observed for any displacement, but can represent the difference between two readings obtained for one complete revolution of the screw. In this case $R$ would be the distance the screw advances in one revolution.

Set the telescope and the scale in front of the lever at a distance of about a meter with both the telescope and mirror at the same height. Adjust the eyepiece until the cross-hairs are clearly defined; then focus the telescope until a sharp image of the mirror can be seen. Sight along the top of the telescope and rotate the stand, holding the lever until the scale is visible. During this operation the image of the mirror in the telescope must remain in sight. Now push in the draw tube of the telescope until an image of the scale appears. If the first attempt is not successful, repeat the above operations with greater care. When the image of the scale has been obtained, move the head from side to side, to see if the images formed by the telescope and by the cross-hairs move with respect to each other. If they do move, the motion must be eliminated; this can be accomplished by altering the position of the eyepiece and then refocusing the telescope. If the image appears to move with the observer, the eyepiece should be drawn out; if the opposite motion is present, it should be pushed in. When the telescope has been properly adjusted, place the screw in its midposition and rotate the mirror in its mounting until the image of that part of the scale which is on a level with the telescope coincides with the cross-hairs. Now turn the screw from its midposition through five complete revolutions and, starting back, take readings for every complete revolution until 10 have been obtained, each reading being estimated to tenths of the smallest division. Thus, there will be readings for five complete revolutions on each side of the midpoint. *Repeat these readings as a check.*

$D$ can be measured by an ordinary meter stick; $P$ by pressing the lever upon a piece of paper and measuring the distance from the imprints. Tabulate the results in five columns; in the first column record the number of the revolutions; in the second, scale readings; in the third, check readings; in the fourth, the average of each pair of corresponding readings; in the fifth, differences between successive readings. In obtaining the average displacement, follow the method described in Par. 10 of the Introduction and do not simply average the differences given in the last column.

### Questions

**1.** Derive the equation by which $R$ is computed and explain why $\frac{d}{2}$ is used.

**2.** How does the distance from the mirror to the scale affect the accuracy of the results? Why?

## EXPERIMENT 5

## The Spherometer

Object: *To measure the radius of curvature of a spherical surface with a spherometer.*

The spherometer is an instrument for measuring very small lengths; its name is derived from the fact that it is usually employed to determine the radii of curvature of spherical surfaces. The essential part of the instrument is a carefully cut screw $C$ (Fig. 8) to which is attached a large disc $D$ which is divided into a large number of equal divisions and can indicate a very small fraction of a turn. This screw is mounted on a tripod, the three legs of which form an equilateral triangle, and, on this same base, is mounted an upright $S$, which serves as an indicator in determining the fraction of a turn of the screw and also carries a scale which denotes the number of whole turns.

In measuring the radius of a spherical surface, the instrument is first placed on a plane surface, and the reading taken when all four points touch the surface. Then a similar reading is recorded when these points are in contact with the spherical surface. If $a$ is the difference between these two readings and $L$ is the length of one side of the triangular base, then the radius can be computed from the formula:

$$R = \frac{L^2}{6a} + \frac{a}{2}.$$

In performing the experiment, first see if the zero on the disc is at the indicator when the edge of the disc is opposite one of the graduations. If this agreement does not exist, ask an instructor to put the instrument in adjustment. Decide in what units the scale is graduated, and determine the pitch of the screw, *i.e.*, the distance the screw advances when it is turned through one complete revolution. Now place the instrument on the plane surface, and turn down the screw until it is in contact, using the knurled disc underneath the graduated circle and not the circle itself. The needle *N* is intended to show when the screw touches the surface. As in all instruments of this type, it is very easy to make mistakes in reducing readings; consequently, the scale and disc readings should be recorded separately and the data tabulated. As an illustration of this reduction, suppose the pitch of the screw is 0.05 in. and there are 500 divisions on the

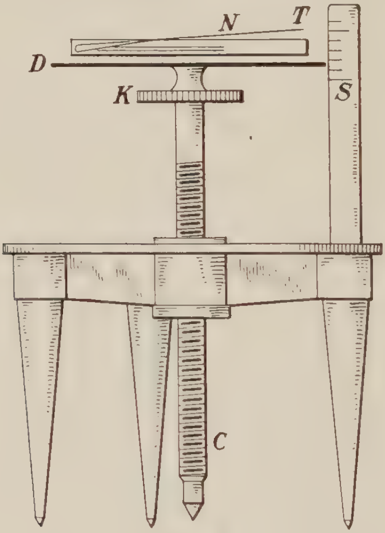


FIG. 8.

disc. In this case each division equals $0.05 \div 500$, or 0.0001 in. For a certain setting, suppose the scale reading to be 2.45 in. and the disc to be at the 235 mark; then the reading is

$$2.45 + \frac{235}{10000}, \text{ or } 2.4735 \text{ in.}$$

Take five such readings on the plane surface. Now place the instrument on the spherical surface and take five more readings. Be sure to turn the screw up far enough so that it is not in contact when the instrument is placed on the spherical surface. Measure $L$ by making an imprint of the three points on a piece of paper and measuring with a steel scale to 0.1 mm. Measure each distance three times. Compute the radius, using the average values of the quantities measured.

### Questions

1. Derive the equation used.
2. Which quantity should be measured with the greater accuracy, $L$ or $a$?

# PART II

# MECHANICS

## EXPERIMENT 6

### Uniformly Accelerated Motion. Falling Body

Object: *To study the motion of the freely falling body; in particular, to determine the distance covered in various times, also the velocity and acceleration.*

*Velocity* is defined as the *time rate of change of position.* Acceleration is the *time rate of change of velocity.* The mastery of these two definitions is presupposed of the student beginning to work this experiment. As regards the distinction between velocity and speed, it will be seen that, since the motion studied here is in a constant direction, the changes of velocity are solely changes of speed, and the terms are here interchangeable.

The measurement of a uniform velocity is relatively easy—requiring merely the determination of the distance covered in a certain time—and its value will not depend on how large an interval of time is chosen. In case of a variable velocity, however, this simple procedure will give only the *average* value for the time measured. The measurement of the true velocity at a certain instant is, in general, attained only by finding the (limit of the) ratio of the distance to the time when the latter becomes vanishingly small.

But there is one notable exception to this difficulty and that is the case where the velocity is changing at a constant rate. Here the average velocity for an interval of time may without difficulty be shown to be the instantaneous velocity at the middle of the time interval taken. Since this case is that studied in the freely falling body, such as used in this experiment, the student should first master this point of relationship between average and instantaneous velocity, before proceeding to the measurements indicated.

As the fall of the plate used in this experiment occupies but a small part of a second of time, the study of the variation of velocity within that interval requires the measurement of times such as $\frac{1}{100}$ sec. For this purpose a tuning fork will be used which makes a known number of vibrations (about 100) per second. The block which is to fall holds a glass plate covered with a film—as thin as possible—of whiting in alcohol. The glass thus coated is fixed to the block and the latter is raised to the top of the guides, where it is held by a pin in the frame. The fork is set into vibration and its stylus is brought into light contact with the plate. The stylus should not press too heavily and it should not chatter in its vibration. The pin is then pulled, permitting the plate to fall. If the adjustments have been properly made, a sharp, sinuous trace will be drawn on the glass. It is best to make several traces side by side; select for use one that is free from sway and has a straight course down the plate. If in doubt which is most promising for measurement, ask the instructor.

*In coating the plate be very careful not to let the whiting be scattered over the apparatus and table. If this happens, it is almost certain that enough will find its way into the guides to ruin (by friction) the accuracy of the experiment.*

## Part I

Before attempting to perform this experiment, read carefully the part of the instruction just after Exp. 63, which applies to the use of an electrically driven fork. Be particularly careful to *pull the switch after the traces have been made.* Neglect of this precaution may result in a ruined fork.

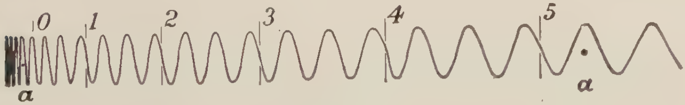


Fig. 9.

The vibrations of the fork occupy equal intervals and may thus be used for marking off small units of time. A suitable interval for a unit of time is that of three complete vibrations of the fork (for a slower fork two). The motion of the plate as it falls past the stylus will be studied from measures of the curve at times arbitrarily called 0, 1, 2, 3, 4, 5, and 6 units. These points are marked on the curve in Fig. 9. As will appear later, it is neither necessary nor desirable to attempt to mark the first point at the instant of starting. Choose the middle of the second or third swing, say, as the zero of time; the (initial) velocity at this time will be computed in Part II.

**Precautions:**

The measures of distance must be made with utmost care, estimating to 0.01 cm., and for this precision the following practice is found best:

1. Near each end of the trace mark as nearly as possible the midpoint of the vibrations ($a$, $a$). These marks may well be beyond the chosen points of measurement.
2. Between these two points lay on edge a *straight* portion of meter stick, so that its metric scale falls along the center of the curve, the rule covering half of the trace. The points to be measured, *i.e.*, where the curve passes under the rule, can be sharply determined. Avoid parallax by looking squarely down along the scale rulings.
3. During the taking of these readings the scale must not be touched or moved. If two persons work the experiment together, the rule should be carefully taken up after one set of readings and replaced for the other student's set, so that another part of the rule is used.
4. *Do not make any marks on the plate which actually touch the trace.* It is essential for exact measurements that the part of the trace which goes under the rule be sharp and clear of any mark.

The data are to be arranged as in the following table and, to simplify explanation, the results of a particular experiment are included. As the tuning fork in this experiment made 135 vibrations per second, the values of the acceleration and other quantities involved will be different from those found with tuning forks actually in use in this laboratory. The italicized figures are the original observations; the others are the deductions.

| Part I | | | | Part II | | |
|---|---|---|---|---|---|---|
| Time | Scale reading | Average velocity (distance per interval) | Acceleration | Total distance | Computed Vo. | Computed velocity at middle of interval |
| Fork units | cm. | cm./int. | cm./int.$^2$ | cm. | cm./int. | cm./int. |
| *0* | *17.21* | | | 0.00 | | |
| | | 1.86 | | | 1.62 | 1.86 |
| *1* | *19.07* | | 0.46 | 1.86 | | |
| | | 2.32 | | | 1.61 | 2.33 |
| *2* | *21.39* | | 0.49 | 4.18 | | |
| | | 2.81 | | | 1.61 | 2.82 |
| *3* | *24.20* | | 0.47 | 6.99 | | |
| | | 3.28 | | | 1.60 | 3.29 |
| *4* | *27.48* | | 0.50 | 10.27 | | |
| | | 3.78 | | | 1.61 | 3.77 |
| *5* | *31.26* | | 0.48 | 14.05 | | |
| | | 4.26 | | | 1.61 | 4.27 |
| *6* | *35.52* | | | 18.31 | | |

Mean acceleration = 0.481 cm./int.$^2$ = 976 cm./sec.$^2$
Average of computed values of $V_0$ = 1.61 cm./int. = 72.5 cm./sec.
Time unit = $\frac{3}{135}$ sec.

The computation of the mean acceleration will be made by the method of Par. 10 of the Introduction. It will not be the simple average of the figures of column 4. Why is this not necessary in handling the value of $V_0$?

## Part II

In performing this experiment it is obviously impossible to begin measurements at the very instant the block starts to fall. It will therefore have acquired a certain velocity $V_0$ at the time the series of measurements is begun, *i.e.*, $t = 0$, and this initial velocity will be computed. The total distance covered by a falling body in time $t$ is given by the formula

$$s = V_0t + \tfrac{1}{2}gt^2$$

and the velocity at any time by

$$V = V_0 + gt.$$

Column 5 gives the total distance covered, and, using the mean value of the acceleration determined in Part I, the series of values for $V_0$ tabulated in column 6 is computed.

One more computation will be of interest and that is for the velocity at the middle of each interval, *i.e.*, $t = \frac{1}{2}, \frac{3}{2}$, etc. If the experiment has been carefully performed, these values should agree fairly well with those in column 3.

**Curves.**—Plot the following curves on one sheet, following the general directions for plotting given in Par. 9 of the Introduction. Use the same scale for the three curves:

1. Times as abscissas and total distances as ordinates.
2. Times as abscissas and velocities as ordinates. In plotting, use the measured values of the average velocity, being careful to choose the correct values of the times for these points.
3. Times as abscissas and accelerations as ordinates.

### Questions

**1.** Interpret these curves, *i.e.*, what relation do they show exists between distance and time, velocity and time, acceleration and time?

**2.** Compute the acceleration and the initial velocity, using the *second* as a unit of time, as was done in the illustration. (See card on fork.) From a consideration of the apparatus, how would one expect the value of $g$ obtained would compare with the true value? Explain.

**3.** How may $V_0$ be determined from the $V$, $t$ curve? Does the result agree with those previously computed?

## EXPERIMENT 7

## Motion of Rotation. Flywheel

OBJECT: *To study a particular case of accelerated angular motion, i.e., to determine the dependence of angular displacement, velocity, and acceleration on time.*

The present experiment is planned to emphasize the similarity existing between motion of rotation and that of translation, and to apply to a revolving body the ideas and facts brought out in Exp. 6.

The apparatus consists of a massive flywheel so mounted that a tuning fork can trace a curve upon the rim; from this curve the data of the experiment are obtained. The wheel is set in motion by means of a weight attached to the rim in such a way that the weight will be disconnected when it strikes the floor. This is accomplished by placing a loop in the string supporting the weight over a little pin set in the rim of the wheel, adjusting the length of the string to make a portion of it still touch the rim when the weight is on the floor. In order to secure the trace, it is necessary to use smoked paper. Fasten the strip of paper to the rim by pasting together the two ends, stretching it as tightly as possible and making the lap in such a manner as not to interfere with the motion of the fork. Use the chemically prepared paper if it is available; otherwise smoked paper. Smoke the paper uniformly with a special gas burner. Do not use the full gas pressure but have a medium-sized, smoky flame. Then wind the string upon the rim until the weight is about 60 to 70 cm. from the floor.

Read the directions following Exp. 63 for using an electromagnetically driven tuning fork. Start the fork and lower it until the stylus touches the paper. Release the weight and, while the wheel is revolving, move the fork laterally, so that the trace of one revolution is not confused with that of the next. Be sure to continue the trace for one complete revolution after the weight has reached the floor. It is entirely unnecessary to make a trace for more than three complete revolutions. In other words, do not shift the fork throughout the entire length of screw, but, as soon as the wheel has made three turns, lift the fork, stop the wheel, and indicate on the trace the point where the weight was removed. This is most readily done by fastening the cord again to the rim and turning the wheel until the weight is just ready to leave the floor. Now move the tuning fork, not vibrating, so that its stylus will trace a line across the paper. Remove the trace as carefully as possible by cutting the paper with a knife at the place where the weight fell off, and, to make the trace permanent, dip it in a weak solution of shellac and hang it up to dry. *Be sure to open the electrical circuit after completing the experiment.* Each student is to make a trace for himself and to include it with his report.

In securing the data, use, as the unit of time, the number of complete vibrations indicated on a tag attached to the instrument; then starting at the point where the force was removed, count back until six units of time have been covered. Consider the point thus determined as zero time for this experiment and then determine the displacement on the rim for twelve units of time, measuring each time the distance covered during that interval and not the total distance, and always estimating to tenths of a millimeter. These readings, of course, give the displacements of the rim during the various intervals of time, but what must be known are the angular displacements of the wheel. With the radius known, the desired data in radians can be obtained. From the data, determine the linear accelerations as was done in Exp. 6 and compute the angular accelerations. In addition, determine the total displacements in radians during the first unit of time, the first two units of time, and so on for all the 12 intervals. The trace obtained for this experiment is to be included in the report.

**Curves.**—Plot all the curves on one sheet.

1. Intervals as abscissas and total angular displacements as ordinates.

2. Intervals as abscissas and average angular velocities as ordinates. In plotting this curve be sure to place the value of the velocity at the instant during the interval when the body is moving with its average velocity.

3. Intervals as abscissas and angular accelerations as ordinates. In obtaining the angular acceleration, the average velocities of the sixth and seventh intervals give a peculiar value. Be sure of its physical meaning before attempting to plot it.

## Questions

**1.** Interpret the curves in detail.

**2.** Compare the curves with those of Exp. 6.

**3.** Explain the peculiar value for the acceleration which results when the average velocity of the sixth interval is subtracted from that of the seventh.

## EXPERIMENT 8

## The Pendulum

Object: *To study the laws of the simple pendulum; in particular, to determine the relation between its length and period of vibration. Also to determine the acceleration of gravitation.*

In the following study of the simple pendulum the work will be divided into two parts: first, a rough attempt to discover the laws of the pendulum by simple and rather crude experimental means; and, second, a more careful, quantitative study of these laws.

### Part I

The student is furnished with a ball fastened to a piece of string. Spend, say, 15 min. in studying this as a simple pendulum. Try to assume the attitude of one who knows nothing about the pendulum, but has a scientific desire to discover the laws which govern its motion. Record in the notebook the observations and conclusions from this preliminary study. **Complete this work before reading further.**

### Part II

The simple pendulum vibrates with a period, *i.e.*, time of complete to-and-fro swing, of

$$T = 2\pi \sqrt{\frac{l}{g}}.$$

It is an example of simple harmonic motion performed along a line which is not strictly straight but, rather, the arc of a circle. The preliminary study should have lead to the conclusion that the vibrations of a pendulum of a given length are *isochronous*, that is, executed in the same time regardless of the amplitude of the swing, provided this is always rather small. This fact makes the pendulum so useful in the escapement of a clock, for it has the same period when the clock is nearly run down as when the spring has been freshly wound up. Prove this isochronous property by experimenting with a pendulum of about a meter length hung from a rigid fixed support. Give it first an amplitude of about 6 cm. (*i.e.*, total swing of 12 cm.) and determine as accurately as possible with a watch the time required for 100 complete vibrations. Do the same thing for an amplitude of about 1 cm. Be sure and count "*zero*" for the first swing which is timed. Why?

The periodic time of the pendulum will now be measured more accurately, and, to do this, the following precautions must be taken: In the first place, it is difficult to determine just when the pendulum reaches the end of its swing, so it is much better to take it when it is passing through its position of equilibrium. Set up a pointer of some sort at this position of equilibrium so that the moment the pendulum is swinging past it may be determined fairly accurately. *Always take such transits in the same direction.* Now, with watch in hand, beat with a pencil upon the table the instants of transit. Record in the notebook the exact time in hours, minutes, and seconds of this the zero transit and also of the fiftieth, hundredth, one hundred-fiftieth, two-hundredth, and two-hundred fiftieth transits in the same direction. Compute the successive differences between readings and determine the periodic time by the method described in Par. 10 of the Introduction. Next measure very carefully the length of the pendulum from the point of support to the center of the ball. Put this in the equation for the period and calculate the value of $g$.

To prove the relation between the period and the length of a simple pendulum, make determinations of the period, counting 100 to 200 vibrations, for three other lengths varying between the original length and about one-fourth of this value. Use small amplitudes in all cases.

**Curves.**—Plot on the same sheet: (1) periodic times as abscissas and lengths as ordinates, and (2) *squares* of periodic times as abscissas and lengths as ordinates.

### Questions

**1.** What sort of curves are the above? What law do they prove?

**2.** At what part of its swing has a pendulum ball its greatest velocity? Its greatest acceleration?

**3.** Would it be expected that a bar, *e.g.*, a meter stick hung as a pendulum would have a longer or a shorter period than a simple pendulum of 1-m. length? Why?

**4.** Determine the period of the large pendulum in the Physical Museum, by timing 30 vibrations, and calculate its length. *Do not under any circumstances disturb its motion.*

## EXPERIMENT 9

## Simple Harmonic Motion

OBJECT: *To study with the aid of data obtained in the foregoing experiment the relation between displacement, velocity, and acceleration in simple harmonic motion.*

Simple harmonic motion is one of the commonest types of motion in nature—exemplified by all sorts of vibratory motions—and its study is, therefore, of considerable importance. It is commonly defined as the projection on a diameter of a point moving with uniform speed in a circle. This point is known as the "point of reference" and the circle is called the "circle of reference." Now it is easily proved (see any textbook of physics) that the acceleration in this type of motion is always towards the center or mean position and that it is, furthermore, *directly proportional to the displacement* but opposite in sign. Sometimes the treatment of simple harmonic motion is just the reverse of the foregoing. That is, one may start with the definition of s. h. m. as a motion in which the acceleration is proportional to the displacement and show that this results from projecting on a diameter uniform circular motion. While the two points of view are slightly different, they lead to exactly the same results.

It may now be seen why the vibrations of a pendulum are isochronous as already found experimentally. Suppose its amplitude is doubled. This means twice as large a circle of reference and also, since the acceleration varies as the displacement, twice the maximum acceleration. Now a point of reference traveling in this larger circle would have twice its original acceleration (as well as velocity) only if it made the circuit in the same time as before, that is to say, the period of the pendulum is unchanged. Throughout this discussion the fact that the pendulum really does not vibrate exactly in a straight line, but in the arc (usually short) of a circle, is neglected.

The motion of the pendulum will now be studied graphically but first the velocity and the acceleration of the point of reference, the projection of which may be thought of as giving rise to the pendulum motion, must be computed. Use the appropriate formulas for a body moving in uniform circular motion, assuming an amplitude of 4 cm. and the period which was obtained for the pendulum about a meter long.

The rest of the computation is to be performed graphically. Construct on a good-sized sheet of paper the quadrant of a circle of 16-cm. radius, that is to say, four times the size of the circle of reference for the motion of the pendulum (which is assumed to be swinging with an amplitude of 4 cm., *i.e.*, a complete swing of 8 cm.). Along the horizontal radius indicate displacements of 1, 2, 3, and 4 units (see Fig. 10), remembering a unit of 4 cm. on this scale is equivalent to 1 cm. on the actual circle of reference for the pendulum. Now erect perpendiculars to the radius at these points, thus determining the position of the point of reference on the circumference of the circle for these various displacements. At these points on the circle draw lines which will indicate the directions of the acceleration and velocity of the point of reference for each of these positions. Then mark off lengths on these lines which will be proportional to the quantities represented.

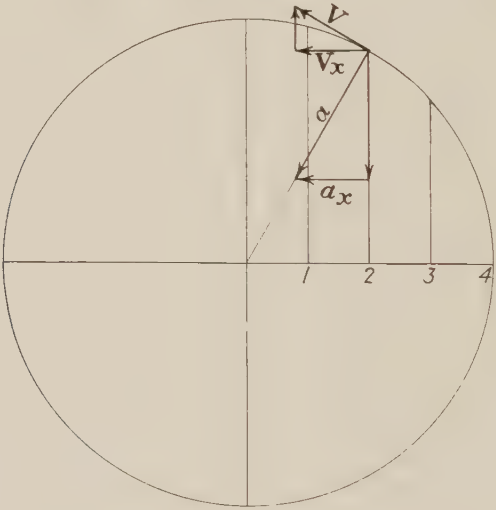


FIG. 10.

The actual velocity and the acceleration of the point of reference have now been represented graphically, but to get the velocity and the acceleration of the point in s. h. m. the horizontal components of these quantities must be taken. From the length of these horizontal lines and by taking into account the scale used, find the numeral values for the actual accelerations and velocities of the pendulum for each of these four displacements. The construction in Fig. 10 shows the velocity $V_x$ and acceleration $A_x$ of the pendulum when displaced 2 cm. from its position of equilibrium.

Determine these values as the point moves through a complete vibration, using displacements from zero to 4 cm. as indicated above. There will then result 16 sets of values for the acceleration and velocity. In the matter of sign, displacements, velocities and accelerations directed to the right are positive, to the left negative. A little thought will show that four careful constructions for each of these quantities will give all these values. Make at least one construction, however, in each quadrant on a small circle to determine the signs of the velocity and acceleration in that quadrant. This latter construction need not, of course, be to scale.

**Curves.**—Plot two curves on one sheet as follows: (1) displacements as abscissas and velocities as ordinates; (2) displacements as abscissas and accelerations as ordinates. As both positive and negative quantities occur in the above, place the origin at the center of the sheet and indicate the values which correspond to a motion of the pendulum to the left by crosses and to the right by small circles.

## Questions

**1.** Explain how a point can move with uniform speed in a circle and still be subject to an acceleration.

**2.** Are the following motions simple harmonic: a hand shake; tiger pacing his cage; piston rod of engine; shadow (from sun in zenith) of handle of revolving grindstone? Give reasons for answers.

**3.** Is there any connection between Hooke's law and the fact that most cases of vibration involve s. h. m.?

## EXPERIMENT 10

## Simple Harmonic Motion. Vibrating Spring

Object: *To study the vibrating spring as a type of simple harmonic motion. To determine from the constant of the spring and its period of vibration the mass of the hanging bob.*

This experiment is meant to emphasize the relation which exists between the acceleration and the displacement for a body moving in simple harmonic motion. It consists of two parts: the determination of the forces exerted by the spring when it is at different distances from its position of equilibrium; then the determination of the periodic time of the vibrating spring and the computation of the accelerations at various points, using a graphic method.

First, find the relation existing between the displacement of the spring and the force which tends to bring it back to its position of equilibrium. To do this, loads, which will be specified on an attached tag, should be placed upon the spring and the corresponding elongations recorded. Of course, the spring should not be vibrating, for what is wanted is the amount the spring must stretch in order for it to exert a certain force. Therefore the readings should not be taken until the spring has come to rest. From these data find the constant of the spring, *i.e.*, the force necessary to elongate it 1 cm. In this computation follow the method given in Par. 10 of the Introduction, and express the constant in c. g. s. units.

The next step is to leave on the spring a specified load, and to allow it to vibrate after it has been displaced 4 cm. from its position of equilibrium. Determine the periodic time by recording the time of the zero transit through the position of equilibrium, then the twenty-fifth, fiftieth, seventy-fifth, one hundredth, one hundred and twenty-fifth, one hundred and fiftieth, and the one hundred and seventy-fifth. Do not count the time of each passage through the position of equilibrium, but only those in the same direction, thus securing data which will give the time of one complete vibration. By the term "zero transit" is meant any passage through the point of equilibrium with which it is convenient to begin counting, and the term is not to be taken in its literal sense. With different springs, different intervals may be more convenient than the ones indicated here; if so, such intervals will be indicated on the tag attached to the springs. Compute the periodic time as described in Par. 10 of the Introduction and include in the data a column showing the successive differences between readings.

The graphic method for the computation of the accelerations can best be explained by referring to Fig. 11. *AC* is the path of the body moving in simple harmonic motion, *O* is the position of equilibrium, and *ABCD* the circle of reference. If the circle is divided into 24 equal parts, then the point of reference will pass over one of these parts in one twenty-fourth of the periodic time. The distance moved by the vibrating body during any one of these intervals may be determined by projecting the corresponding arc onto the diameter *AC*. For example, during the second interval the point of reference moves from *J* to *K* and the body moves from *E* to *F*. This distance represents the average velocity for the second interval. The acceleration for the displacement *OF* is obtained by subtracting the average velocity of the second interval from the average velocity of the third interval. This is strictly true only for uniformly acceler-

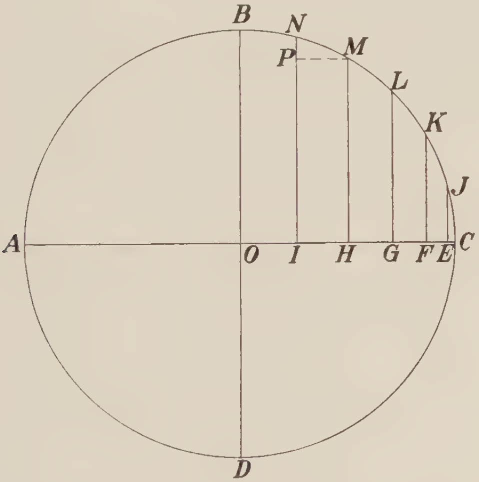


FIG. 11.

ated motion, but the acceleration is changing slowly enough so that this approximation may be made.

As large and as accurate a drawing as possible should be constructed. Otherwise, unsatisfactory results will be obtained. Use a large sheet of paper, which will be furnished at the desk, and construct on it a quadrant of a circle, the radius of which is some multiple of four. Divide the arc into six equal parts and drop the perpendiculars as indicated in Fig. 11. Now measure the average velocities for each of the intervals. This can be done more accurately if the distance corresponding to *MP*, for example, is measured instead of the actual distance *IH*. Express these velocities in centimeters per second, taking into account that the actual circle of reference has a radius of 4 cm. Compute the accelerations for each of the displacements.

The accuracy of the work is to be checked by computing the mass of the vibrating body from the data of the experiment. Take a certain displacement, as *OG*, and compute the force exerted when the spring tends to bring the body back to its position of equilibrium. This can be done, as the constant of the spring is known. Be sure to use the actual displacement and not the one measured directly from the drawing. The acceleration for this position has just been determined; consequently, the mass can readily be computed, use being made of Newton's second law. Is the variation between this result and the bob mass due entirely to experimental error?

**Curves.**—1. Elongations as abscissas and loads as ordinates.

2. Times as abscissas and accelerations as ordinates.

3. Displacements as abscissas and accelerations as ordinates.

### Questions

1. Interpret the curves.
2. What simple test for s. h. m. is indicated by the data of this experiment.
3. Show how the facts brought out by the curves serve as an experimental proof that the spring was vibrating in s. h. m.

## Descriptive Note

**Fletcher's Inclined Plane and Trolley Apparatus.**—The Fletcher apparatus is so designed that a large number of experiments illustrating many of the fundamental principles of mechanics may be performed with it. Figure 12 shows the general arrangement. It consists of a metal track *A*, 7 ft. long, grooved so that a car *B* can run along the surface with very little frictional resistance. The car consists of a strong wooden block, mounted on wheels, with holes bored in it so that weights of a special form can be securely carried, thus enabling the mass moved to be varied through considerable limits. The forces acting on the car are obtained either by inclining the plane, which allows a component of gravity to act or by placing weights of various sizes in the holder *E*. To measure the velocity or acceleration, a curve is traced by a flexible vibrating strip *F* on a sheet of paper attached to the upper surface of the car. This strip is supported by the frame *C*, the position of which may be changed. The frequency of vibration depends upon the point at which it is clamped, and, as the determination of the frequency is a tedious task, care should be taken not to alter this length. The trace is made by a small camel's hair brush attached to the end of the strip. The brush is inked by a quill fastened to the stopper of the bottle, and, to obtain a satisfactory trace, the ink should be applied so that the brush forms a single sharp point.

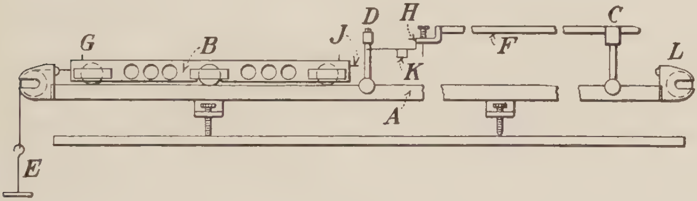


Fig. 12.

The following procedure will be found advantageous in making a trace: The car should be placed at the end of the track where it will be at the close of the run. After the brush has been inked, the car is drawn back under the stationary brush to its position for starting the run. This operation should result in a sharp straight line being drawn on the paper. To obtain this, the brush must be not only at the proper height but the paper must also be stretched so as to form a perfectly smooth surface. If the trace shows that these adjustments have been properly made, the strip may be set in vibration and the actual curve obtained. When the work is completed, the ink left in the brush should be carefully washed out.

In the following experiment it is necessary to have the car start just as the strip makes its first vibration. This is accomplished by a starting device attached to the support of $D$. $H$ is a strip of metal mounted so as to rotate about the vertical axis and provided with a projection $K$ which engages the pin $G$. The portion of this strip which extends beyond the part $K$ is bent at right angles forming a vertical projection which the strip strikes when it starts its first vibration, thus releasing the car. The distances between the supports $D$ and $C$ can be adjusted so that the release can take place without disturbing to any appreciable extent the vibration of the strip. In order to obtain satisfactory results, the position of the vibrating strip when at rest must be such that it is just on the point of releasing the car.

## EXPERIMENT 11

## Newton's Second Law of Motion

Object: *To measure the acceleration produced in a given mass by a given force and to compare it with that calculated from Newton's Second Law of Motion.*

One of the simplest and most instructive problems involving Newton's second law of motion is that of the acceleration of a car which rolls with as little friction as possible on a nearly level surface and is attached to a vertically falling mass by a string passing over a pulley. Fletcher's apparatus (read preceding description) will be used in this connection. It should be kept in mind in performing the experiment that the car and the falling mass constitute one dynamic system and both move with the same (numeric) acceleration. The force acting on the system is, accordingly, the *weight* of the falling mass, while the total mass moved is the sum of the two. To eliminate, so far as possible, the effects of friction, the track is given a slight downward slope, so that the car, when given a small impulse, will run from end to end at a nearly uniform speed. The experiment consists of two parts, both of which are required; first, the resulting accelerations will be determined when a constant force is applied to different loads; and, second, the accelerations produced by different forces acting on the same mass will be measured.

### Part I

To obtain the constant force for this part of the experiment, a certain mass (see card on apparatus) will be placed on the holder $E$ and kept there throughout the run. The force of gravity will exert a constant pull on this mass and holder, no matter how they move, and this, their weight, is the force which is moving the system. A trace is to be made for each of the loads for the car, indicated on the card. Be sure that the vibration strip releases the car just as it crosses the reference line which it made when at rest. The acceleration is then readily computed from the total distance covered in several vibrations. Use a complete vibration as the unit of time, and, as the accuracy of the result is greater the larger the length measured, choose as the distance that corresponding to the total number of complete vibrations in the trace. *Do not change the adjustment of the vibrating strip in any way,* as this would alter its period and necessitate a difficult and tedious redetermination.

### Part II

To secure a variable force and at the same time keep the mass constant, the load when taken from the car, piece by piece, will be transferred to the pan. A rack $J$ (Fig. 12), is fastened to the trolley to hold the small weights. Traces are to be made for each force and the acceleration measured as in the first part.

Tabulate the results of both Part I and II as follows:

| A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|
| Car mass $M$, in grams | Hanging mass $m$, in grams | Total mass, $M + m$ | Force $mg$, in dynes | Observed acceleration, in cm./interval$^2$ | Observed acceleration, in cm./sec.$^2$ | Computed acceleration | Computed tension in string, in dynes |
| | | | | | | | |

In computing column F from E, make use of the period of the vibrating strip measured in seconds as given on the card. The tension in the string may be readily computed if it is recalled that it is this pull which is accelerating the mass $M$. It may also be figured in another way: If the system is at rest, the tension in the string is evidently merely the weight of the hanging mass, *i.e.*, $mg$. If this mass were to be raised with an acceleration $a$, the tension would have to be increased by the amount $ma$, *i.e.*, $mg + ma$. Similarly, if, as in the present case, the mass were to be lowered with the acceleration $a$, the tension would be reduced to $mg - ma$. Compute the tensions by either method, but in at least two instances use *both* methods to see if they check. Use as accelerations those computed in column G.

**Curves.**—The curves are to be plotted on two separate sheets.

Sheet 1, Part I

1. Masses $M + m$ as abscissas and observed accelerations as ordinates.
2. *Reciprocals* of masses as abscissas and accelerations (not reciprocals) as ordinates. Use table of reciprocals at end of manual and carry the value of the accelerations to only three significant figures.

Sheet 2, Part II

Applied forces as abscissas and observed accelerations as ordinates.

### Questions

**1.** Distinguish carefully *mass* and *weight.*

**2.** Interpret the curves. Are they in agreement with Newton's second law? Explain. Interpret any intercepts if they exist. (See Par. 9 of the Introduction.)

**3.** Suppose the holder and hanging masses weigh 100 gm.; compute the tension in the cord (in dynes) in the following cases: (a) system not moving; (b) system moving with acceleration $\frac{1}{5}$ $g$; (c) system moving with constant velocity $v$.

### *Optional*

**4.** Outline a dynamic method of measuring the mass of the car, knowing the mass of the hanging body and the acceleration produced in the system.

## EXPERIMENT 12

## Tension in a Cord Attached to a Body Moving with Accelerated Motion

OBJECT: *To compare the observed and calculated values of the tension in the cord which is lifting a body with accelerated motion.*

From Newton's second law of motion the conclusion may be reached that the force accelerating a body is measured by the product of the mass and the acceleration and produces its own effect independently of any other system of forces acting on the body. Thus, if a mass $m$ hangs on a cord, the tension in the cord is $mg$ dynes, provided there is no acceleration. If the body is pulled up with an acceleration $a$, this tension will have to be increased by the amount $ma$. Likewise, if it is lowered with an acceleration, the tension will be diminished by a similar amount.

In the present experiment this tension will be measured. The apparatus consists of a bicycle wheel mounted on a support so as to be several meters from the floor. The masses $M$ and $m$ (Fig. 13) are attached to a long string which passes over the wheel; $m$ is the smaller mass and includes the mass of the balance which is just above and which is hung upside down. This is provided with two hands, one of which is moved by the mechanism of the balance, while the other is free, but has sufficient friction to hold it in any position in which it may be placed. A projection on the latter engages the first hand, and so the two move together for increasing loads, but, as soon as the load is removed, the first hand moves back, leaving the other at the maximum reading. If $M$ is raised nearly to the limit of its motion and the system held at rest, the balance will indicate the weight of $m$, provided the restraining force is applied to the cord at the point $d$ (see Fig. 13). As soon as the system is released, the force, or tension in the cord, when the system is subject to acceleration, will be registered.

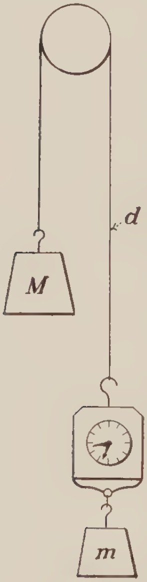


Fig. 13.

To obtain the best results, certain precautions must be observed. The system must be released as suddenly as possible; this is most easily accomplished by holding the cord with only the thumb and forefinger. It is equally important to stop the motion before $M$ hits the floor, and this should be done as gradually as possible. As the desired effect is due to acceleration, it is necessary for the system to move freely for only a few centimeters, so there should be no trouble in stopping the system in time. The balance hand used to indicate the maximum reading does not always work as it should, so several trials without touching it between times should be made until it indicates the largest reading. It may now have gone too far, and, to test this, set it back an ounce at a time until it is moved up again when $M$ falls. Take this last position as the reading. Hence, to secure a single reading, $M$ must be raised and allowed to fall several times, for, unless this is done, most unsatisfactory results will be obtained. Determine tensions for the loads indicated on a card attached to the apparatus.

The balance reads in pounds and ounces, and so the observations must be reduced to grams' weight. The balance itself is usually somewhat inaccurate, and the maximum indicator slightly out of adjustment. The simplest way to correct these errors, and at the same time to transform the reading, is by means of a calibration curve. After the experiment is finished, attach the balance to a fixed support, and record the reading of the indicator when 1.2, 2.2, 3.2, and 4.2, kg.-wt. have been added. Then plot a curve and from this change the readings into grams' weight. It will probably be found most convenient to reduce the ounces to decimal parts of a pound. It will be remembered that $m$ includes not only the masses added, but also the mass of the principal part of the balance. This can be obtained by reading the position of the indicator when the balance is supported by the hook.

When the tension for the various loads has been measured, check the accuracy of the experimental work by computing what the tension should be. To do this, remember that the acceleration of the system as a whole will be *the force acting on it* divided by the *total mass moved.* Now in this case the total mass moved with an acceleration $a$ is the *sum* of the masses $M$ and $m$, and the total force producing this accelerated motion is the *difference* of the weights of these masses (expressed in dynes). The acceleration is thus readily obtained and the tension can be computed as suggested in the first paragraph of this discussion. *Tabulate the results.*

**Curve.**—Abscissas, weights applied to calibrate balance; ordinates, corresponding readings of the balance in pounds and fractions of a pound.

## Questions

**1.** Distinguish *mass* and *weight.*

**2.** Imagine a man on a spring balance in an elevator. State whether his apparent weight would have its normal value or be greater or less than this under the following conditions: (*a*) elevator rising with an acceleration $a$; (*b*) elevator standing still; (*c*) elevator descending with a constant velocity $v$; (*d*) elevator descending with an acceleration $a$; (*e*) elevator rising but slowing down; (*f*) elevator descending but slowing down; (*g*) elevator falling freely with a broken cable.

**3.** Show how the above would be changed if, instead of a spring balance, he were standing on an ordinary lever balance using various weights as counterpoises.

### *For Engineering Students*

**4.** Answer the above questions quantitatively, assuming the man's weight to be 120 lb. and the acceleration $a$ as 8.05 ft./sec.$^2$

## EXPERIMENT 13

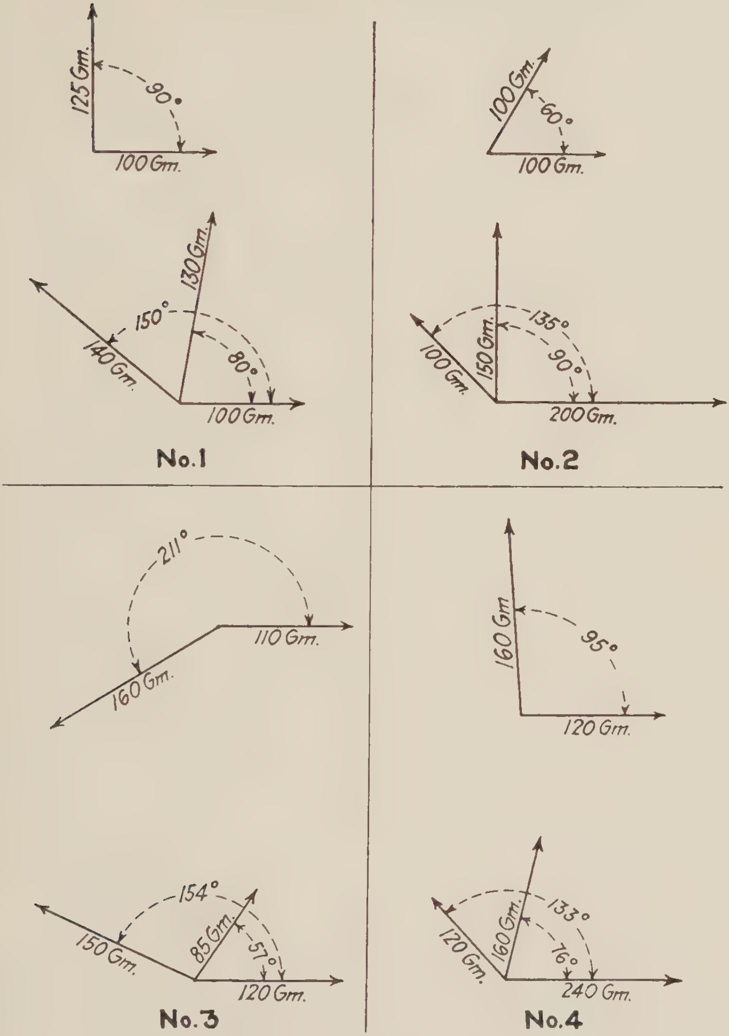
## Equilibrium of Non-parallel Concurrent Forces

Object: *To determine graphically and also experimentally the resultant of several non-parallel forces.*

If several non-parallel forces are acting at the same point on a body, it can be shown that a single force properly applied will neutralize the other forces and the body will be in equilibrium. The magnitude and the position of this anti-resultant may be obtained by the parallelogram of forces, and a study of this principle will be made in the present experiment.

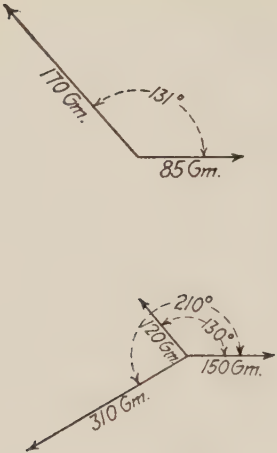
The apparatus used consists of a circular table graduated in degrees and provided with movable pulleys which may be clamped in any position on its circumference. A string passes over each of these pulleys, one end of which is attached to a ring at the center of the table and the other end to a weight holder, thus making it possible to apply forces of any desired magnitude. The experiment consists in finding the resultant for two known forces, and then for three, particular problems being given when the experiment is assigned Lines, the direction and the length of which specify the known forces, are drawn either directly upon the table or on paper laid on the table; the parallelogram is constructed, and the direction and the magnitude of the resultant are obtained from the proper diagonal. Great care must be observed in constructing the figure in order to get good results.

When the anti-resultant has been obtained, test the accuracy of the work by setting up the proper forces at the proper angles on the table and seeing if they are in equilibrium. Determine in radians the angles which are formed by the three forces in equilibrium. Do this by actually measuring the radius of the table and the arcs which subtend the various angles. After each problem is completed and the forces are in position on the table, ask an instructor to inspect the work. Record on the data sheet the magnitude of the forces in equilibrium and the angles between them. *Redraw the figures on plotting paper, using one sheet for each figure and making the drawings as large as possible.* It is unnecessary to reconstruct the parallelogram, for protractors will be furnished so that the angles found on the force table may be reproduced on a sheet of plotting paper.



No. 1 No. 2 No. 3 No. 4

Problems for Exp. 13.



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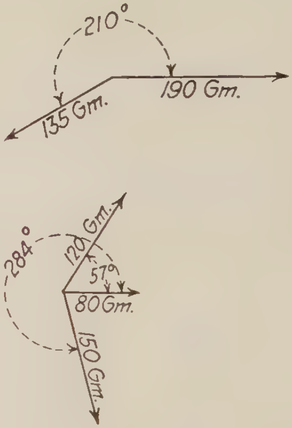

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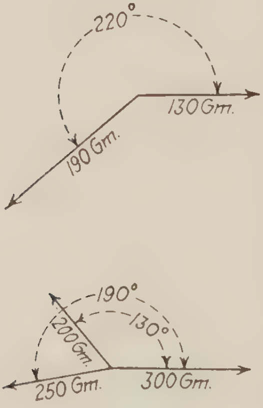

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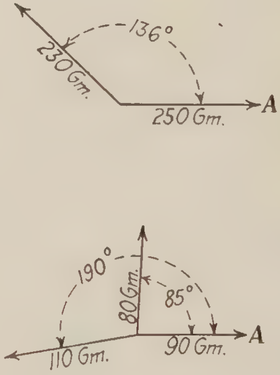

**No. 8**

Problems for Exp. 13.

A simple way to test the forces acting all in the same plane, to see if they are in equilibrium, is to resolve each force into two components along two perpendicular axes. The sum of these components along each axis is zero if the forces are in equilibrium. Apply this method to the first problem, *i.e.*, the one which has the two given forces, and see that the components are added along each axis separately and that each set is shown to reduce to zero. Another important test is that forces in equilibrium may be represented in magnitude and direction by the sides of a closed polygon. Using this, see if the second problem will check. Start by drawing a line parallel and equal to the line representing one of the forces. At the end of the line draw another which represents the direction and magnitude of the force adjacent to the first one. At the end of this draw a third one, and so on. If the forces are in equilibrium, the figure will close. Indicate in the report how closely the drawings satisfy these tests, showing in the first test the actual magnitudes of the components in kilogram-weights, and in the second by how much the figure fails to close.

### Questions

**1.** Define resultant, anti-resultant, radian.

**2.** A man weighing 180 lb. sits in a hammock, the ropes of which make angles of 40 deg. with the horizontal. Determine graphically the tension in each rope.

**3.** A picture weighing 20 lb. is hung by a cord which passes over a nail and fastens at each side of the picture frame. If the cord can, at most, support a weight of 15 lb., find graphically the largest angle it can make with the vertical.

## EXPERIMENT 14

## Parallelogram of Forces. The Derrick

Object: *To study the composition of forces as illustrated in the derrick.*

The parallelogram of forces has numerous applications in everyday life. In this experiment the derrick, one of the illustrations most used in textbooks, will be studied.

In its simplest form the derrick consists of an upright portion called the post or mast; attached to this is a boom or jib which can be raised or lowered by a suitable tackle called the tie. At the end of the boom is another tackle which supports the load. The complete study of all the forces acting in or on the various parts of the derrick would make a long and involved experiment: accordingly, the work will be limited to the forces acting at the point $O$ (Fig. 14*a*). In the experimental derrick the boom is of special form, so designed that it measures the thrust exerted by the boom, and a spring balance determines the force in the tie. If the load is attached as indicated in Fig. 14*a*, there will be practically three forces acting at $O$, for the tensions in the loop supporting the load will have a resultant directed downward and equal to the load. When arranged as in Fig. 14*b*, the conditions will be similar to those in actual practice and four forces will be acting at $O$. In performing the experiment, both of these cases are to be studied, a load of 10 kg. being used in the first case and a load of 5 in the second. The experimenter is to select the angles at which the various forces are to act, but such ones should be used as will give fairly large readings on the instruments. The forces indicated by the spring balances are to be read and the angles which the various forces make with the vertical are to

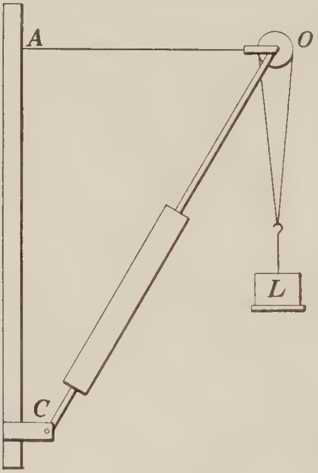

Fig. 14*a*.

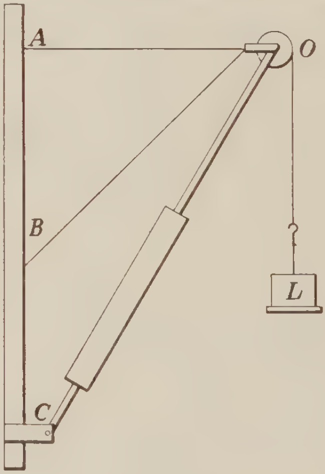

Fig. 14*b*.

be determined. This last measurement is most conveniently obtained by means of a special form of protractor. Before the readings are taken, it will be found advantageous to displace the boom by a very small amount and allow it to come back to its position of equilibrium, thus tending to reduce the errors due to friction. Tapping the boom gently during this operation will also assist materially.

When the data have been obtained, parallelograms are to be constructed, the sides of which are to be the forces exerted by the load and the tie. From these the force exerted on the boom is to be determined and the computed and observed values compared. In the first case the problem is to be looked upon as one of three forces in equilibrium; the second case is that of four forces. The tension in the cord attached to the mast at point *B* in the second problem is to be considered equal to the load. The tests described in the last paragraph of Exp. 13 are also to be applied to these problems.

It is interesting to study the way in which the forces in the various parts change as the angles at which they are working are varied. First use the arrangement shown in Fig. 14*a*. Do not change the load *L* or the point of attachment *A* of the tie. Vary the angle of inclination of the boom through its limit of motion and observe the change in the magnitude of the thrust exerted by it. Then test the change produced when four forces are used, starting with the cord *OB* attached at some convenient point, which is to be kept constant. The point of attachment of the tie *A* is to be moved up as far as possible and the balances read when the boom is nearly vertical, and then again when it is horizontal. The point of attachment of *A* is to be shifted as far down as possible and readings again taken for both positions of the boom.

### Questions

**1.** Why is the compression in the boom for the second arrangement of about the same magnitude as in the first, although the load is half as great?

**2.** State the best position for the boom and the attachment of the tie in Fig. 14*b*:

(*a*) If the tie is in danger of being subjected to a force greater than it can exert.

(*b*) If the boom is likely to have too great a load placed upon it.

## EXPERIMENT 15

### Equilibrium of Parallel Forces. Couples.

Object: *To determine the resultant of parallel forces.*

Since a rigid body has two forms of motion, translatory and rotational, two conditions must be fulfilled in order that it may be in equilibrium. One of these is that the resultant of all the forces acting must be zero, and the second is that the sum of the moments of the forces about any point must be zero. In the present experiment parallel forces will act upon a body and, by the use of the conditions of equilibrium, the position and the magnitude of the anti-resultant are to be determined.

The rigid body to be used is a long, rather narrow, rectangular piece of wood, which is counterbalanced so that its weight may be neglected. It is placed in a frame provided with stops which allow the bar freedom of motion within narrow limits. Forces may be applied to this bar by hanging weights upon it, and, by the use of mounted levers, some of the forces may be made to act upward. The positions of the forces are determined by a scale marked on the bar; they are all in the plane of the bar.

Special directions will be given which will specify the several forces whose anti-resultants are to be found. First see that the bar is counterbalanced and that the sliders on it are made of wire or string, the weight of which may be neglected. *Compute the magnitude, the direction, and the point of application of the anti-resultant and then test the result on the apparatus.* To find the magnitude and the direction of the anti-resultant, make use of the first condition of equilibrium; determine the point of application by the second condition.

#### Questions

**1.** State the conditions of equilibrium of a rigid body and explain why two are necessary.

**2.** Define moment of force. To what is it numerically equal?

**3.** Define couple. Why cannot a single force hold it in equilibrium?

**4.** Draw conclusions as to the position of the opposing couple, which holds a given one in equilibrium. If the bar were brought to equilibrium by the application of a couple consisting of two forces of plus and minus 3 kg.-wt. applied at 20 and 60 cm. respectively, would equilibrium still be maintained if these forces were applied at 50 and 90 cm.?

**5.** Explain why the computations could be made just as well if the zero mark were at the center of the bar instead of at one end.

## EXPERIMENT 16

## The Whiffletree

Object: *To study the compositon of parallel forces as illustrated by the whiffletree.*

The whiffletree offers many problems of special interest. The one selected for this experiment is the variation in the forces exerted by the horses as the whiffletree makes various angles with the tongue.

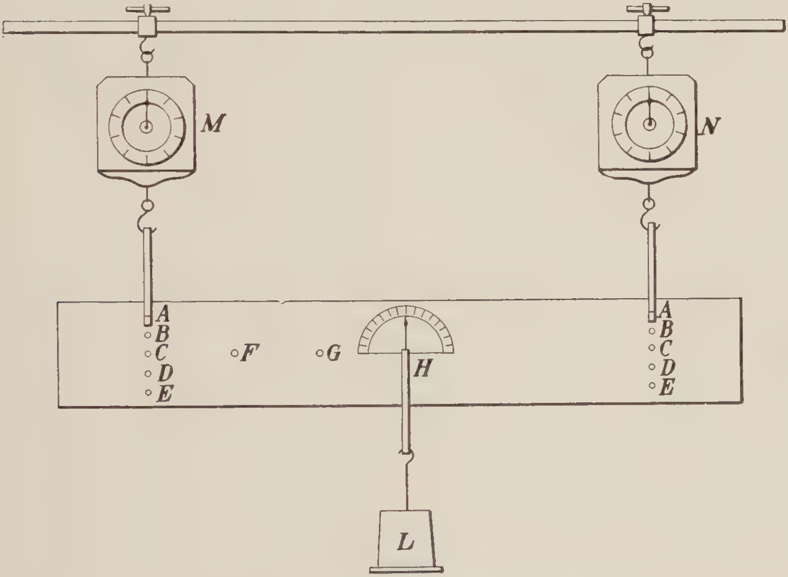

Fig. 15.

The apparatus is shown in Fig. 15. The whiffletree is represented by the board *CHC*. The forces exerted by the horses are indicated by the spring balances *M* and *N*. *L* represents the pull exerted by the wagon and its load. The apparatus is so constructed that the direction and the points of application of the forces *M* and *N* may be changed as desired.

The following observations are to be made:

1. Attach balances to the holes *AA* and a load of 5 kg. at *H*. Record readings of balances when the evener is inclined at the angles 0, 10, 20, and 30 deg. Before determining the values of

the angles, shift the attachments of the balances so that they are acting vertically, and see that the various clevises are swinging freely. It is not necessary that the angles be exactly those indicated, but measure carefully the magnitudes of those used.

2. Repeat observations, but attach balances at *CC*; then at *DD*; finally at *EE*. In each position record the four readings indicated in observation 1.

3. When the balances are in the holes *EE*, shift the upper attachments of the balances so that the forces are no longer acting in parallel lines. Record readings of the balances for two different positions, moving the attachments outward each time. Is it possible by this adjustment to make the two balances read the same?

4. Have the whiffletree horizontal and place *N* at *C*. Read both balances as *M* is shifted to *C*, *F*, and *G*.

**Curves.**—Plot curves for the data taken for the different angles of inclination of the whiffletree with the forces applied at the various points.

### Questions

1. Interpret the curve, giving reasons why the balance readings should vary as they do.
2. Compute what the forces exerted by the balances should be for one of the experimental arrangements in observation 4.
3. Explain why the balance readings in observation 3 change as they do.

## EXPERIMENT 17

## The Principles of Equilibrium Applied to a Rigid Body. The Fixed-arm Derrick

OBJECT: *To study the equilibrium of forces and moments of force in a particular form of derrick.*

In order that a body may be in equilibrium, two conditions must be satisfied: The vector sum of all the forces must be zero, and the sum of the moments about any point must likewise be zero. These conditions existed in Exp. 15 with parallel forces, but in this experiment non-parallel forces will be used; any rigid body might be taken, but a small, fixed-arm or bracket derrick will be employed, as it is an interesting illustration. The experiment consists in a study of the external forces acting on the derrick, and no attempt is made to analyze the forces present in the different parts of the derrick itself.

The apparatus is arranged as shown in Fig. 16. The derrick is provided with a sharp point at its base, which is set into a metal disc. In obtaining the data, apply at *B* a load, which will be designated on a tag, and measure by the spring *A* the force necessary to produce equilibrium. When this reading has been obtained, shift the positions of the load and the spring and repeat, using the same weight at *B* as in the first case. These two sets of readings constitute the data of the experiment. Be sure to measure the distance *c* and *d* each time and be very careful in making the adjustments that the derrick is vertical and that the spring is horizontal. Determine the mass of the derrick and its center of gravity. Is it necessary to determine the center of gravity of the derrick or merely the vertical line passing through the center of gravity?

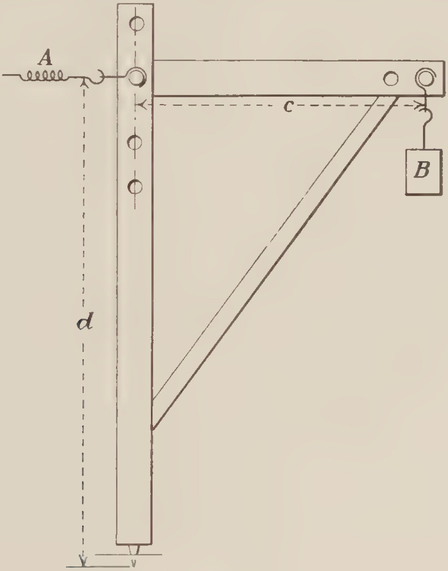

FIG. 16.

There are four forces acting—the load *B*, the force indicated by the spring *A*, the weight of the derrick, and the reaction at the base. Three of these are known; find the fourth in each set by the polygon of forces. Knowing all the forces acting, find the moments in each set, first about the base and then about any point selected at random not in the line of action of any of the other forces. In the last computation the magnitudes of the lever arms are to be obtained by drawing the derrick to scale and determining the distances graphically. These drawings should be made as large as possible.

### Questions

1. Why does a different number of forces enter into the second computation of each set than into the first?

## EXPERIMENT 18

### Centrifugal Force

Object: *Comparison of the measured and the computed forces involved in central acceleration.*

A careful examination of the apparatus (Fig. 17) renders very little description necessary. The speed of rotation is controlled by the rheostat until the hook $H$ on the bob $B$ just clicks the slip of cardboard $P$ at each revolution. This slip must have a clean-cut edge. The rheostat will probably have to be adjusted several times during the course of the run to keep the instrument at just the right speed, and such adjustments should, in general,

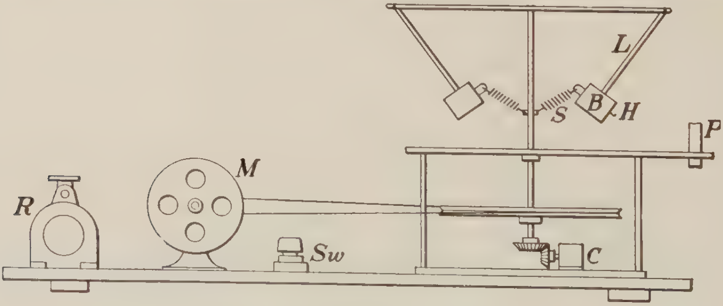

Fig. 17.

be very small, that is, over only one or two turns of wire. When running at the right speed the hook will *barely touch* the paper as it goes by. If the belt becomes loose or breaks, replace it with a new one, stretching the cord, however, before it is put on.

After becoming fairly familiar with the apparatus and making a few short trial runs, set the little cardboard slip so that, when the hook just strikes it, the bob hangs down vertically below its point of support. Start the apparatus running and, after it has settled down to a fairly constant speed, take readings of the revolution counter each minute for 10 consecutive minutes. Treating these by the method described in Par. 10 of the Introduction, find to two decimals the number of revolutions per second. Reduce this to radians per second and then compute from the formula $CF = M\omega^2R$ the centripetal force which must be exerted by the spring on the bob. This involves the measurement of the radius, which is to be taken from the center of the axis to the center of the bob. In computing this centripetal force, the mass of the spring and also of the small suspending rod must be taken into consideration. It can be shown that these are accounted for by adding to the mass of the bob (which is stamped on its top), *one-half* that of the rod and *one-third* that of the spring. The rod weighs 8.3 gm. and the spring 11.0 gm.

This computed force is to be compared with that necessary to stretch the spring by the same amount as occurs under running conditions. There are two ways of measuring this: The best method is to run a horizontal cord from the little hook over a pulley and down to a pan on which various weights may be placed. In this way, if the pulley is well oiled, a very good determination may be made of the necessary force. The simpler way is to use the spring balance furnished with the apparatus. In either case, be very careful that the force is applied exactly along the line of the spring. If the balance is used, read the upper part of the pointer and make a correction of 20 gm. because it is used in a horizontal position.

Make two complete runs of the experiment.

**Do not speed up the apparatus faster than it is intended to be run. The student will be held strictly responsible for any accident which may result from carelessness in this respect.**

### Questions

**1.** Show why one-half of the mass of the rod should be included with that of the bob.

**2.** Distinguish between *centripetal* and *centrifugal* forces. On what does each act and in what direction when the apparatus is run?

**3.** What are the units of force involved in the expression $M\omega^2R$?

## EXPERIMENT 19

## Elasticity. Determination of Young's Modulus

OBJECT: *To determine Young's modulus for a wire of given material.*

One of the most important of the constants of elasticity is Young's modulus, and its determination forms the principal part of this experiment. It is most easily obtained by applying forces to a wire and measuring the resulting elongations. In this type of elastic deformation the modulus is obtained from the following equation:

$$M = \frac{Fl}{qe},$$

where $M$ represents Young's modulus, $F$ the force applied, $l$ the total length of the wire, $q$ the cross-section of the wire, and $e$ the elongation. Accordingly, the data of the experiment consist in finding the length and the cross-sectional area of the wire, and the elongation due to the force $F$.

The elongations of the wire produced by the forces are so small that the changes in length have to be measured as carefully as possible. For this purpose a small comparator will be used. Before starting to take any readings, study the construction of the instrument carefully, so that the method of using it is thoroughly understood; 15 min. spent in mastering it may mean the saving of several hours. A description of the comparator will be found following Exp. 27. The indicator, upon which the cross-wires of the microscope are to be set, is a cylinder with the upper edge at right angles to its axis. If this edge is illuminated from below by an electric light, it forms a sharply defined object very suitable for this purpose. Equally good results are obtained if the lower edge is lighted from above. If the wire does not show too great a tendency to twist as it stretches, a fine horizontal scratch will serve very well as an indicator.

Find the elongations of the wire for the loads indicated on a tag attached to the apparatus. Do not measure the elongation produced when the first load is added, for that one is intended to straighten the wire, but record the reading of the comparator after the load is added. Record on the data sheet the readings of the instrument for the various loads; in doing this, it is well to indicate separately the readings of the disc and the scale, and then the two combined, so as to be able to detect possible mistakes in computing. In the tabulated data show the elongation produced by each increment of load, *i.e.*, the difference between successive readings. Also include the total elongation or the difference between each reading and the first one. The measurements should be taken not only for increasing but also for decreasing loads. Make 10 determinations of the diameter of the wire, using a micrometer caliper and measuring at a number of different places; also find the length of the wire. Find the average elongation due to the increment of load which is added each time, using the method described in Par. 10 of the Introduction. It will probably be necessary to make separate determinations, using first the readings obtained with increasing loads and then those with decreasing ones. Thus, all the quantities needed to find Young's modulus are known. Determine it in c. g. s. units and also in grams' weight per square millimeter.

**Curve.**—Total elongations as abscissas and loads as ordinates. Indicate by crosses the elongations produced by increasing loads, and by dots surrounded by circles those due to decreasing loads.

## Questions

**1.** Interpret the curve, *i.e.*, mention all the physical facts that it shows.

**2.** Define elastic limit. Was it exceeded at any time during this experiment? What part of the data shows this?

**3.** Show how the equation for $M$ may be derived.

**4.** Can any modification of the experiment be suggested which eliminates the error due to the yielding of the support to which the wire is fastened?

## EXPERIMENT 20

## Power

Object: *To determine the power output which a person can develop under certain conditions.*

The apparatus consists of a large hand wheel (Fig. 18), on which a simple form of Prony brake has been rigged. For the first trial, place, say, a 5-kg. mass on the holder, which should be on the right side of the wheel. Run the cord from the holder over the wheel, through the pulley and up to the 15-kg. balance, which is to be held against the wall and about 4 or 5 ft. high. Start turning the wheel (left-handedly) while the holder and the weights are still resting on the floor and the cord is slack; then pull up the balance until the weight and the holder are lifted perhaps a foot, the wheel meanwhile being rotated at a uniform speed of 60 to 100 r. p. m.

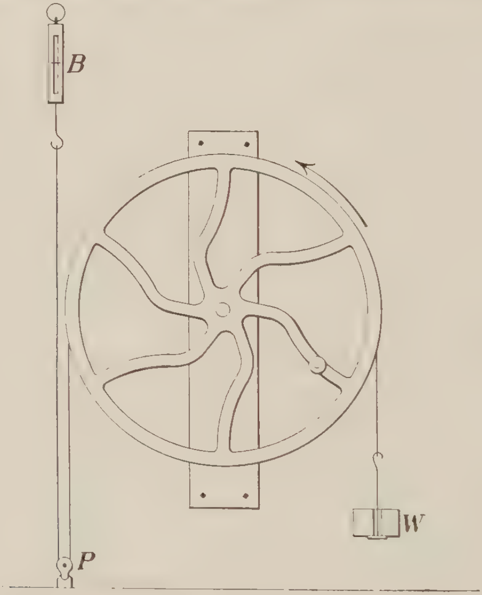

Fig. 18.

A convenient way to measure the r. p. m. is to have the student who holds the balance call "Tick!" at the beginning and again at the end of a 10-sec. period (the watch may be conveniently hung on the little hook for this purpose) while the one at the wheel counts the revolutions. Be sure to begin to count with *zero*.

The indications of the balance are somewhat in error because of the friction of the pulley. To eliminate this, raise slowly and again lower the balance as the wheel is being turned, and take the mean of the two readings as the correct one.

Make *four* runs on the experiment, two for each student at the wheel. In one of these two, choose such weights and speed that the student feels he could maintain this rate of work for some time—20 min., say—without undue exhaustion. This gives his *continuous* power rating. In the second run place 15 to 20 kg. on the holder and turn the wheel as fast as possible for the 10 sec. This will give the maximum (short time) power development. Express results in *watts* and also *horsepower*.

### Questions

**1.** Develop and explain the formula used.

**2.** A 150-lb. man runs up stairs at such a rate that he rises 4 ft. vertically per second. What horsepower does he develop?

**3.** In clearing a 6-ft. pole a jumper raises his body 4 ft. and requires only $\frac{1}{5}$ sec. for the take-off. If he weights 160 lb., what horsepower does he develop momentarily?

**4.** A powerful horse weighing 1800 lb. leaps forward (from rest) acquiring a velocity of 20 miles per hour in $\frac{1}{2}$ sec. What is his (instantaneous) power development?

## Descriptive Note

**Simple Machines.**—A simple machine is a device by which a force applied at one point is changed into a force acting at another point which may differ in both direction and magnitude from the first. The force exerted by the machine usually overcomes the friction of the various parts and, if the machine is for lifting purposes, the weight of the body moved. This last, in the laboratory, consists of weights of known magnitudes and will be called the load. The energy which enables the machine to move is furnished by the applied force, which likewise consists of weights. When the statement is made that a simple machine is capable of transforming a small force into a large one, it does not mean that such a device furnishes more work than is supplied it, for the larger force always moves through the smaller distance, and, in a perfect machine, the work which it does should always be equal to the work done upon it. In a discussion of simple machines the following terms are frequently used:

The *velocity ratio* is the ratio existing between the distance moved over by the applied force and that covered by the load. In a perfect machine, it is equal to the mechanical advantage, and in an actual machine it is often spoken of as the theoretical mechanical advantage.

The *mechanical* advantage is the ratio between the load and the force necessary to move it. In practice this ratio is always found to be less than the velocity ratio.

The *mechanical efficiency* is the ratio of the useful work accomplished by the machine to the work performed upon the machine. This quantity cannot be greater than 100 per cent.

## EXPERIMENT 21

## Efficiency of a Simple Machine

OBJECT: *To determine the efficiency and the mechanical advantage of a simple machine.*

This experiment answers the question as to what effect friction has upon the efficiency of a simple machine and upon the forces necessary to operate it. Before starting to do any experimental work, read the preceding note on simple machines. The instructor will assign as apparatus either the wheel and axle or the block and tackle, but the same general instructions will apply to both.

Find the applied force which is necessary to raise the load with uniform motion for each of the loads given on a card attached to the machine and then the one necessary to lower it with the same motion. The average of these two is the applied force required to move only the given load, the force used to overcome friction having been eliminated. It is sufficient to measure these forces to 20 gm., *i.e.*, if 850 gm. is too large, try 830 gm., not 845 or 840. An attempt at accuracy greater than this is a waste of time. When the data have been obtained, compute the efficiency for each load when the machine is raising that load, for then it is working under the usual conditions.

To obtain this efficiency, it is necessary to find the velocity ratio or the relative distances moved by the load and the applied force. This can most readily be done by placing the moving force at a known distance from the floor and, after it has passed over that distance, determining how far the load has been raised. Do this for three distances but for only a single load, as it should be the same for all loads. Determine the efficiencies for the loads used in obtaining the data and also for the loads of 0.5 and 1 kg., obtaining the necessary data for the two latter from curve 1. These forces have to be found from the curve, since the friction for small loads is too irregular for accurate experimental work.

**Curves.**—Select such a scale that a large curve will result, but have the point corresponding to zero load and zero applied force at the lower left-hand corner of the page. Plot all the curves on one sheet.

1. Loads as abscissas and applied forces necessary to raise the load as ordinates.
2. Loads as abscissas and average applied forces as ordinates.
3. Loads as abscissas and theoretical applied forces as ordinates. These are forces which would be used if the machine were perfect, and they can be computed, since the velocity ratio is known.
4. Loads as abscissas and efficiencies as ordinates, including the values corresponding to loads 0, 0.5, and 1.0 kg.

*For Engineering Students*

5. Loads as abscissas and mechanical advantages as ordinates.

### Questions

**1.** Compare curves 1, 2, and 3, explaining the intercepts and differences in the slopes, if any exist. (See Par. 9 of the Introduction for an explanation of these terms.)

**2.** State what the fourth curve shows, giving a physical explanation.

**3.** Explain why the method employed eliminates the force necessary to overcome the friction.

**4.** Why is it necessary to adjust for uniform motion?

*For Engineering Students*

**5.** Interpret curve 5.

## EXPERIMENT 22

### Moment of Inertia of a Rigid Body. Compound Pendulum

OBJECT: *To determine the moment of inertia of a rigid body by measurement of its period when swung as a compound pendulum.*

When a body is of irregular shape its moment of inertia cannot be computed. Some experimental method must then be employed and one of the most convenient is to mount the body in such a manner that it can vibrate as a compound pendulum. The expression for its period is then

$$T = 2\pi\sqrt{\frac{I}{mgh}},$$

where $T$ represents the periodic time, $I$ the moment of inertia, $m$ the mass of the pendulum, $g$ the acceleration due to gravity, and $h$ the distance from the point of support to the center of gravity of the system. All these quantities can readily be determined except $I$.

The object whose moment of inertia is to be determined is a metal ring. Suspend it from a knife edge, set it vibrating through a small angle, and determine its period by noting the time of the zero passage through the point of equilibrium and then of the fiftieth, one hundredth, and so on for every fiftieth transit until the three hundred fiftieth is reached. Use only swings in the same direction and determine the time of a complete period just as in Exps. 8, or 10, employing the method outlined in Par. 10 of the Introduction. Measure $h$ and determine the mass of the ring by weighing.

The moment of inertia is then readily obtained with the aid of the above equation for the periodic time. This gives, however, the moment of inertia about the axis through the point of support. To find it about the center of gravity of the figure, the formula $I_a = I_0 + ma^2$ must be used where $I_0$ is the moment of inertia about the center of gravity, $I_a$ the moment of inertia about a point at a distance $a$ from the center of gravity, $m$ the mass of the body.

Compare this experimental value of $I_0$ with that computed from the fundamental definition. As the thickness of the ring is small compared with its radius, the mass may be considered, in this computation, as concentrated at a distance from the center equal to the average radius.

Also suspend the ring on a knife edge so as to swing in a direction perpendicular to the plane of the ring. From the period obtained in this way compute the moment of inertia about a *diameter*. Compare with the other two moments of inertia.

### Questions

**1.** Define radius of gyration and determine it for both of the values $I_0$ and $I_a$.

**2.** How could this method for the determination of the moment of inertia be applied to a mounted flywheel?

## EXPERIMENT 23

## Moment of Inertia of a Flywheel

OBJECT: *To determine the moment of inertia of a flywheel from measurements of the work done in setting it in rotation and its resultant angular velocity.*

If the amount of work that is used up in setting a body in rotation and the resulting angular velocity of the body are known, then the moment of inertia can be computed, since the kinetic energy which it possesses is a function of its moment of inertia and angular velocity. This is a brief statement of the method of determining the moment of inertia of the flywheel used in Exp. 7. The wheel is set revolving by a known weight falling a known distance; thus the energy furnished the system can be computed. This energy is used up in putting the wheel in motion and in imparting kinetic energy to the falling weight. Expressing this symbolically,

$$mgd = \tfrac{1}{2} I\omega^2 + \tfrac{1}{2} mV^2,$$

where $m$ represents the mass of the known weight, $d$ the distance it has fallen, $\omega$ the resulting angular velocity of the flywheel, $V$

the velocity of the weight at the end of the fall, and $I$ the moment of inertia of the flywheel.

To obtain the data for the experiment, attach the weight to the wheel by a cord and have it so arranged that, when the weight touches the floor, the cord will be detached. Wind the cord about the wheel until the weight is at a known distance from the floor, and let it fall, being careful that no impulse of any kind is given it when it starts. The most satisfactory way of doing this is to hold the wheel in place after the weight is wound up by means of a string fastened to two pins, one in the frame and the other on the wheel. Then the wheel may be released at the proper time by burning the string. To measure $\omega$, fasten to the rim a piece of chemical paper or smoked paper and with a tuning fork of known frequency make a trace on the paper, observing the various precautions mentioned in Exp. 7. Be sure to break the circuit through the tuning fork when the work is completed. For a few turns after the weight has stopped, the wheel will move with uniform angular velocity and, from the trace, can be determined the velocity with which the rim is moving; with the radius of the wheel known, $\omega$ can be easily computed. It is obvious how the final velocity of the weight is to be found. Thus, all the quantities in question are known except $I$, so that can be determined.

A common rule for an approximate computation of the moment of inertia of a flywheel is to neglect the hub and spokes and consider only the rim, assuming the mass of the rim as concentrated at its mean radius, thus giving conditions which will allow the use of the fundamental definition of the moment of inertia in the computations. Apply this rule to the flywheel used in this experiment and determine the mass of the rim from its volume and density. The rim is made of cast iron and has a density of 7.1 gm. per cubic centimeter.

## Questions

1. Explain any discrepancies between the two values.
2. Derive the formula used.
3. Disregarding friction, which of the following would roll down an incline most rapidly: (*a*) a hoop; (*b*) a solid disc; (*c*) a block on light wheels? Give reasons.

## EXPERIMENT 24

### Coefficient of Friction

OBJECT: *To determine the coefficient of friction for two particular surfaces.*

When an attempt is made to cause two bodies to move over each other with uniform velocity, a certain force is required to overcome the friction between the two surfaces. It has been found that, if this tangential force is divided by the normal force, the quotient obtained is constant throughout considerable variations in force, and this constant is called the coefficient of friction.

A way of determining it is to have one of the substances in the form of a plane placed horizontally with a pulley fastened at one end. The other substance is in the form of a block, to which is attached a cord passing over the pulley and carrying weights that are varied until the block moves uniformly when once put in motion. The normal force between the surfaces can be changed by placing different weights upon the block itself, and the statement that the coefficient is independent of the normal force may thus be tested. Find the force necessary to move the block uniformly when 2 kg. are placed on it, *making two independent determinations*, and trying to maintain as nearly as possible the same uniform velocity. Repeat with loads up to 14 kg., increasing by 2 kg. at a time. Be careful not to touch the surfaces, as moisture will affect them. Compute the coefficient for each load.

**Curves.**—1. Normal forces as abscissas and moving forces as ordinates.

2. Normal forces as abscissas and corresponding values of the coefficient of friction as ordinates.

#### Questions

**1.** What relation exists between the force of friction and the force applied parallel to the plane?

**2.** Why is it necessary that the body move with uniform velocity?

**3.** A problem is required which will be assigned by the instructor reading the student's notebook.

## EXPERIMENT 25

## Principle of Archimedes. Specific Gravity

Object: *To determine the specific gravity of a solid heavier than water, of one lighter than water, and of a liquid.*

In doing this experimental work, great care must be observed not to disarrange the various parts of the balance and thus throw it out of adjustment. The main thing to guard against is a displacement of the knife edges supporting the pans. If these are not in their proper places, all readings taken under these conditions are valueless. *Be sure to follow the directions given at the end of Exp.* 3 *and to record on the data sheet the zero of the balance at the beginning and at the end of the experimental work, together with masses determined.*

First work with the substance whose specific gravity is greater than unity. Find its weight in air by suspending it by a piece of fine wire from the hook just beneath the beam of the balance. Then immerse the body in distilled water by placing a beaker on a small bench which will support the vessel over the pan of the balance but still in no wise interfere with the free motion of the latter. After all air bubbles which tend to cling to the body have been removed, and the body is so adjusted that it does not touch the beaker, weigh it again. Compute its specific gravity.

It is obvious that the same method is not directly applicable to the body lighter than water, but a little thought will show how the difficulty may be overcome by combining it with the heavier body and weighing both together in water. The student is asked, if possible, to reason this method out for himself.

The specific gravity of a liquid other than water is found by weighing the heavier solid in it and comparing its loss of weight in this liquid to that in water.

Make two independent determinations in each case.

### Questions

1. Distinguish between density and specific gravity.
2. State the principle of Archimedes. Show its application in finding the specific gravity of a substance.

3. Draw diagrams showing the vertical forces acting on the two bodies when they are immersed in water and indicate the magnitude of these forces.
4. Is the accuracy of the result dependent on the amount of water used?
5. A beaker of water and a small cylinder of brass are placed side by side on a balance pan and the weight of the whole determined. The cylinder is now placed in the beaker and, by Archimedes' principle, obviously now exerts a smaller downward force on the bottom of the beaker than it did on the pan outside. What will be the effect on the weight of the system? Show how the answer is in agreement with Newton's third law of motion.

## EXPERIMENT 26

## Boyle's Law

OBJECT: *To verify Boyle's law.*

In order to specify the condition of a gas, it is necessary to know its pressure, volume, and temperature. These three quantities are connected by a well-known equation, but in this experiment a simpler form of this equation will be considered, that is, the relation existing between the pressure and the volume when the temperature is constant. This relation is usually called Boyle's law.

The gas is confined in a vertical glass tube, while parallel to it is an open tube capable of being moved up and down. These are connected by rubber tubing and are partially filled with mercury which traps the gas. By raising and lowering the open tube, the height of the mercury column may be varied, and thus the pressure changed. The pressure is determined by the difference in heights of the two mercury columns and is measured by a metric scale placed between the two tubes. This difference must, of course, be added to or subtracted from the barometric reading to find the total pressure. The volume of the gas is to be measured in cubic centimeters from the graduations on the tube. The particular pressures which are to be used will be found on a tag attached to the apparatus. *Be sure to read the barometer.* Find the product of the volumes and the corresponding pressures.

In making the computations, use only three significant figures in the numbers representing volume and pressure, and carry the product to four figures. The table of reciprocals at the end of the book will greatly lessen the work of computing.

**Curves.**—1. Volumes as abscissas and pressures as ordinates.

2. Volumes as abscissas and reciprocals of pressures as ordinates.

### Questions

**1.** Interpret the curves.

**2.** Should the products of the pressures and the corresponding volumes be a constant? Explain.

# PART III

# HEAT

## EXPERIMENT 27

### Coefficient of Linear Expansion

OBJECT: *To determine the coefficient of linear expansion of a metal.*

The equation of linear expansion in its simplest form is

$$l_t = l_0 (1 + at),$$

where $l_0$ is the length at 0°C., $l_t$ at $t$°C. and $a$ the coefficient of expansion. Solving this equation for $a$, it is seen that, to determine this quantity, the original length, together with the change in length produced by a given change in temperature, must be known. As the coefficient is small for metals, it will be allowable, for the purpose of this experiment, to take $l_0$ as the length at the lower of the two temperatures, and $t$ as the difference between this and the higher.

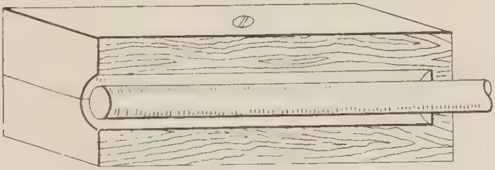

FIG. 19.

The metal, in the form of a thin walled tube, is fastened rigidly to a wooden base by a clamp of the sort shown in Fig. 19. It will be noted that this grips the tube firmly at just one definite point. The change in length is to be measured by a comparator. Before going further with this experiment the student should read over the accompanying description of this instrument and familiarize himself with it. When the comparator has been mastered, this instrument is to be clamped firmly on the wooden base and adjusted so that a scratch on the surface of the tube is in sharp focus and appears at the intersection of the cross-hairs of the eyepiece. Care must also be taken that the direction of travel of the microscope is parallel to the length of the tube.

Cold water is now run through the tube until a thermometer inserted at one end shows a constant temperature. Be careful that this thermometer is fastened with a bit of wire, so that it cannot be carried out of the tube by the running water. Three successive readings of the comparator are now taken and the temperature noted in degrees and tenths; then, without disturbing either tube or microscope in the slightest degree, change the hose connection and run steam through instead of water until the thermometer is again constant. In making this change disconnect the hose at the glass connection near the end of the tube. In measuring the temperature, the thermometer is to be inserted so that only the end of the mercury thread is exposed. If the flame is under the boiler for some time, be very careful that it does not boil dry.

Having taken three readings at steam temperatures, return to the cold-water arrangement and take three more readings, which will be averaged with the first measurements. The coefficient of linear expansion can now be computed after the length of the tube has been measured. This may be done with a common meter stick, measuring from the stop to the scratch on the tube.

Make two independent determinations of the coefficient in this way, using different lengths of tube.

### Questions

**1.** Show by rewriting the expansion equation that the coefficient of linear expansion is the change of length per degree per centimeter of the original length.

**2.** Why must the increase in length of the tube be measured so carefully while the length itself can be determined with an ordinary meter stick?

**3.** Could this approximation involved in calling $l_0$ the length at the lower temperature (say 8°C.) be made if the coefficient were equal to 0.005?

## Descriptive Note

**The Simple Comparator or Traveling Microscope.**—This instrument is in general use for the accurate measurement of short distances. Figure 20 shows the form used in this laboratory, consisting of a low-power microscope *M* which slides smoothly along carefully made guides or ways. The screw which produces the motion has been accurately cut to some convenient pitch (in this case ½ mm.) and the amount of displacement is measured by the number of whole turns and fraction.

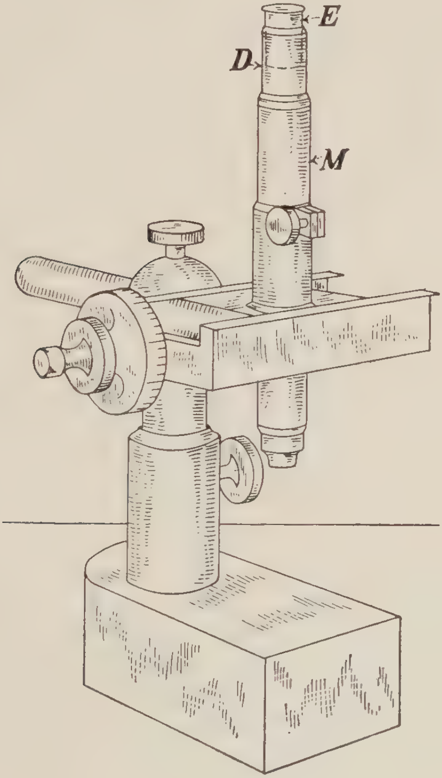

Fig. 20.

In using the instrument, the eye piece of the microscope must first be adjusted so that a sharp image of the cross-hairs is formed;

the whole instrument is then focused on the point under observation and, if the adjustment is good, there should be no parallax, *i.e.*, the image and the cross-hairs should not move with respect to each other when the eye is moved slightly from side to side. If parallax is found, it must be eliminated by further adjustment of the eyepiece and refocusing the microscope.

The first precaution to be taken in using such an instrument is to avoid errors due to *lost motion.* It will be noticed that the screw may be turned back and forth through a good many divisions without producing any accompanying motion of the carriage, because the screw is purposely made to fit very loosely in the nut (see Fig. 21) and it is obvious, then, that readings must always be taken *with the screw turned in the same direction.*

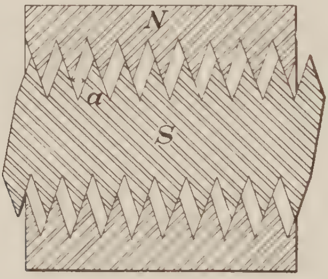

Fig. 21.

Whether this is to be right- or left-handedly is readily decided by noting which way gives the closest check of the zero on the micrometer head with an even division of the scale. When, in taking a reading, the screw is turned too far, run it backwards half a turn or more and again approach the setting from the same side as before. In manipulating the instrument do not handle the micrometer head $D$ but always turn the screw by means of the knurled disc $K$ (Fig. 22).

It is very easy to make mistakes in reducing readings, so the instrument and its scale must be carefully studied and thoroughly understood before starting an experiment. The scale is divided into centimeters and tenths, *i.e.*, millimeters, and the micrometer head $D$ into 100 equal parts. Since it takes *two complete turns* of the screw to move the carriage a millimeter, each division on the micrometer head has a value of 1/200 mm. Obviously, then, if the reading of the micrometer head be divided by 2, the result will be expressed in hundredths of a millimeter. This indicates the simplest way in which to read the instrument. First read the scale directly in millimeters, *e.g.*, 14 mm. in Fig. 21. To this add half the reading of the micrometer head, *e.g.*, 48/2 = 24 hundredths of a millimeter. The reading, then, is 14.24 mm. It must be carefully noted in this connection that the reading of the micrometer head will be exactly the same if the screw is advanced one turn further, but the difference will be evident in the reading on the scale, for the index mark will then be *more than half way* past a whole division. If this were true in the present case, the reading would be 14.50 + 0.24 = 14.74 mm. Unless careful attention is paid to this point, the results are likely to be in error by one whole turn, *e.g.*, ½ mm. In general, after completing a reading, reduce it to centimeters. Since readings are to be made to a tenth of a division, *i.e.*, to 1/2000 mm., the result in centimeters should contain four places of decimals, *e.g.*, 1.5823 cm., 2.6900 cm., etc.

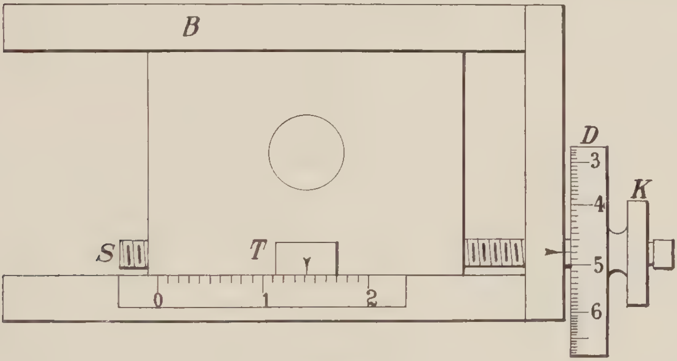

Fig. 22.

## EXPERIMENT 28

## Pressure Coefficient of a Gas

OBJECT: *To determine the pressure coefficient of air at constant volume.*

The law of Charles may be stated in two ways, depending upon the conditions under which the gas is heated. Thus, if the pressure is kept constant, the volume of the gas increases; if, on the other hand, the volume is constant, the pressure must increase. Both conditions are represented in the mathematical forms of this law, as the following equations show:

$$P = P_0 (1 + a_p t) \text{ volume constant.}$$
$$V = V_0 (1 + a_v t) \text{ pressure constant.}$$

For experimental purposes it is more convenient to use the change in pressure at constant volume and this method will be employed in this experiment. The gas whose coefficient is to be determined is air and is contained in a large glass bulb attached to a long, slender tube. On this tube is etched a ring which marks the limits of the constant volume. In order to apply a variable pressure, another glass tube, open to the air, is attached to the bulb by a rubber tube and filled with mercury. Both of these tubes are fastened to vertical rods along which they may slide; a meter stick is mounted between them. The method of measuring pressure is obvious if the barometric pressure at the time the readings are made is known.

To perform the experiment, lower the open tube until the mercury is 5 or 10 cm. below the etched ring; immerse the bulb in a copper beaker, filled with water at room temperature or lower, being sure that the bulb is entirely submerged. Wait a few minutes; then adjust the mercury columns until the column on the closed side is just at the ring; read the position of the two columns and finally measure the temperature of the water. By careful application of a flame the water can be raised to, and held at, various other temperatures. The simplest way to read the height of a mercury column is to place a piece of paper which has a straight edge so that this edge is parallel to the graduations on the two sides of the meter stick and is also on a level with the top of the meniscus. If two observers are working at once, let one adjust the mercury columns and read the position of the column in the open tube at the same time that the other observer is reading the temperature. Then immediately lower the mercury column so that it is 5 or 10 cm. below the etched ring. Repeat these operations for the various temperatures until six readings have been made. The last reading should be that of boiling water and the various readings should be separated by as nearly equal intervals as can conveniently be obtained. In all this work care should be taken to prevent the mercury from passing over into the bulb, because the data obtained after that will be worthless. To guard against this, always lower the mercury levels as soon as a reading is taken and, when the last observation is made, lower the open tube until the mercury is below the first reading observed. *Be sure to record the barometric reading and the reading on the meter stick corresponding to the etched ring.*

**Curve.**—Temperatures as abscissas and corresponding pressures as ordinates. In plotting temperatures, start with 0°C. at the lower left-hand corner of the sheet; it is not advisable to start at 0 cm. in plotting pressures, but begin with a value which will allow the curve to intersect the axis of ordinates above the origin. The $P_0$ is the pressure at 0°C. and none of the above readings gives this value. To find it, extend the curve just described until it cuts the ordinate corresponding to 0°C. This intersection gives the value of $P_0$. Be sure to take into account the scale used in plotting pressures. Having obtained $P_0$, compute $a_p$ for the various readings.

### Questions

**1.** Explain the way in which $P_0$ is obtained.

**2.** What unconsidered errors enter into this experiment, and how do they affect the result?

**3.** Why should the introduction of mercury into the bulb during the experiment make the data worthless?

## EXPERIMENT 29

## Gas Thermometer

OBJECT: *To acquire a knowledge of the principles and the use of the constant-volume gas thermometer.*

Since all gases have, within certain limits, the same coefficient of expansion, the use of gas as a thermometric substance is only reasonable. While it cannot be called a practical instrument from the standpoint of everyday use, the gas thermometer, suitably corrected, is the basis of the temperature scale and it becomes of great importance as an instrument of calibration.

Although hydrogen and nitrogen are the best gases for this purpose, for the sake of simplicity (dry) air will be used. The apparatus is the same as that used in Exp. 28 and the student is asked to read the directions to be followed in its use.

If a change in temperature from 0° to 100°C. produces a change in pressure from $P_0$ to $P_{100}$, then the change per degree is

$$\frac{P_{100} - P_0}{100}.$$

At some other temperature $t$, a pressure $P_t$ would then be observed such that

$$P_t - P_0 = \frac{t}{100}(P_{100} - P_0)$$

or

$$t = 100\frac{P_t - P_0}{P_{100} - P_0}.$$

This indicates the way in which the instrument may be used to measure temperature. $P_0$ and $P_{100}$ are the pressures at the freezing and boiling points of water. The boiling point will not be exactly 100°. Calculate just what it is from the barometer reading and *use this value instead of 100* in the preceding equations.

Make two complete runs, determining in each case $P_0$, $P_{100}$, and $P_t$ at some intermediate temperature $t$. Calculate this temperature (it need not be the same in the two runs) and compare with the reading of a mercury thermometer.

### Questions

1. Give two sources of error in this experiment and indicate how they might be eliminated.
2. What difficulties attend the use of a *constant-pressure* gas thermometer?

## Descriptive Note

**Calorimetry.**—Calorimetry is the science of heat measurement. Measurements of heats of combustion, latent heats of fusion and vaporization, and specific heats of various substances are among the problems of this branch of physics. The fundamental principle underlying all the work is that the amount of heat furnished by one component of the system (*e.g.*, a piece of hot metal) must be equal to that absorbed by the other (*e.g.*, the calorimeter). A discussion of calorimetry and of the equations which may be used in calculating results will be found in any physics text, but it is far more instructive for the student to build up his own equation. Find the portions of the system which are furnishing heat and those which are absorbing it. Express these quantities of heat in terms of the masses, temperature differences, and specific heats of the materials. If the work has been correctly done, there will be but one unknown quantity in the resulting equation (*e.g.*, specific heat, latent heat of fusion, etc.).

*Water Equivalent.*—The ideal calorimeter would have a massless cup for holding the water. As this is not realizable we must take account of the heat absorbed by the metal of the cup as well as by the contained water. The easiest way to do this is to compute the water equivalent of the calorimeter cup, that is, the mass of water which would require the same quantity of heat to raise its temperature through one degree centigrade as is required by the calorimeter for this purpose. Obviously this is the mass of the calorimeter cup multiplied by the specific heat of the material of which it is made. Having found this water equivalent it is generally most convenient for purposes of computation to add this quantity to the mass of the water in the cup.

*Sources of Error.*—These are principally as follows: (1) inaccuracy in the determination of the various masses involved;

(2) errors in temperature measurement; and (3) radiation losses. The first source of error is obviously to be eliminated by careful weighing, special care being given to the determination of the amount of water in the calorimeter. Errors of thermometry are not as easily corrected, for the ordinary laboratory thermometer is graduated only in degrees, and frequently the total change of temperature does not exceed 10°. Obviously, then, it is necessary to read thermometers as carefully as possible, *estimating in all cases to one-tenth of a degree.* In certain experiments thermometers graduated to one-fifth or even one-tenth of a degree will be furnished, in which case readings should be made to one-fiftieth or one-hundredth of a degree respectively.

The third source of error arises from the fact that a vessel at a higher temperature than its surroundings is constantly losing heat, while the reverse is true if it is at a lower temperature. The term "radiation losses" used in this connection is really a misnomer, for convection currents in the air surrounding the calorimeter are largely responsible for the loss, while some heat is conducted to the jacket surrounding the calorimeter and only a small part is dissipated by direct radiation. The simplest method of correcting for these losses is that due to Rumford and consists in arranging the experiment so that the calorimeter at the beginning of the experiment is as much below room temperature as it is above it at the end. It will then be seen that the gain from radiation, conduction, etc. during the first half of the experiment will be approximately compensated for by the loss during the second half. A little thought will show, however, that this compensation is by no means perfect, for in the first half the calorimeter is rising in temperature more rapidly than in the second half; consequently, it will be above room temperature more than half the time and the radiation loss will exceed the gain. To correct this, it is usual to start at a temperature a little below that indicated above.

A somewhat more complicated but much more accurate method of correction is due to Regnault and may be described as follows: It is an experimental fact, known as Newton's law of cooling, that the rate at which heat is given off by a cooling body is directly proportional to the difference in temperature between it and its surroundings. This law is only an approximation, but holds with sufficient accuracy for ordinary calorimetric work for a temperature difference of 30° or less. It also applies when the body is cooler than its surroundings, in which case heat is

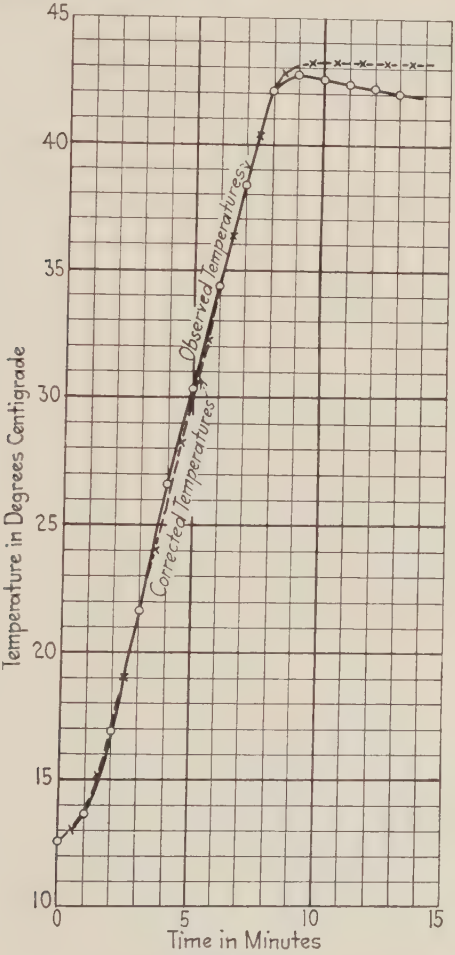

Fig. 23.

absorbed instead of radiated. The usefulness of this law depends upon the fact that, if the rate at which the body is cooling can be determined for one temperature, *e.g.*, when the calorimeter is warmest and hence cooling most rapidly, it is at once known for all others within the above limited range. In Fig. 23 the solid-line curve gives the actual temperatures found in a particular experiment. In this particular case the calorimeter was surrounded by a water jacket at 24°C. This would then be taken as "room temperature." The last half dozen points determine a straight line and from the slope of this line the rate of cooling can be determined. Thus, at the end of the tenth minute the temperature is 42.6° and at the end of the fourteenth, 41.9°. Hence, the loss per minute is 0.175° for an average temperature of 42.25°. This point is, accordingly plotted, as in Fig. 24. A second point can obviously be determined from the fact that when the calorimeter reaches room temperature, *i.e.*, 24°, its radiation loss will be zero. By drawing a straight line through these points a curve is obtained which will be used in making corrections.

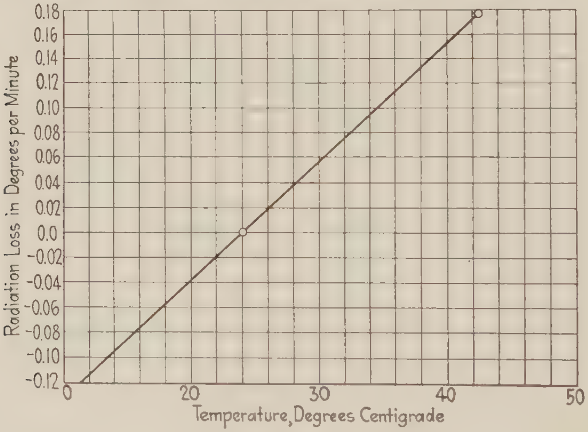

FIG. 24.

The way in which this is done will appear from an inspection of the table below. The rate of cooling (or heating) is determined for the average temperature in each interval and the sum of these separate corrections gives the total correction. The corrected curve is plotted as a broken line in Fig. 23. It shows at the end a horizontal straight line or constant temperature showing that the cooling is correctly allowed for. In this particular case the difference between the original and the corrected curve is small because the cooling in the latter half of the experiment is partly compensated by the heating in the early part. If the initial calorimeter temperature were that of the water jacket, the corrected curve would lie above the other for its entire length. It will be noted that in the above application of Newton's law of cooling quantities of heat lost are not being actually dealt with, but rather temperature changes, which in this case are proportional to these quantities.

| Interval in minutes | Degrees Centigrade | | | | | |
|---|---|---|---|---|---|---|
| | Initial temperature | Final temperature | Mean temperature | Cooling correction | Total correction | Mean temperature corrected |
| 1 | 12.6 | 13.6 | 13.1 | −0.10 | −0.10 | 13.0 |
| 2 | 13.6 | 16.9 | 15.25 | −0.08 | −0.18 | 15.07 |
| 3 | 16.9 | 21.7 | 19.3 | −0.04 | −0.22 | 19.08 |
| 4 | 21.7 | 26.6 | 24.15 | 0.0 | −0.22 | 23.93 |
| 5 | 26.6 | 30.3 | 28.45 | 0.04 | −0.18 | 28.27 |
| 6 | 30.3 | 34.4 | 32.35 | 0.08 | −0.10 | 32.25 |
| 7 | 34.4 | 38.4 | 36.4 | 0.12 | 0.02 | 36.42 |
| 8 | 38.4 | 42.1 | 40.25 | 0.15 | 0.17 | 40.42 |
| 9 | 42.1 | 42.75 | 42.42 | 0.18 | 0.35 | 42.77 |
| 10 | 42.75 | 42.6 | 42.68 | 0.18 | 0.53 | 43.21 |
| 11 | 42.6 | 42.4 | 42.5 | 0.18 | 0.71 | 43.21 |
| 12 | 42.4 | 42.25 | 42.32 | 0.18 | 0.89 | 43.21 |
| 13 | 42.25 | 42.05 | 42.15 | 0.17 | 1.06 | 43.21 |
| 14 | 42.05 | 41.9 | 41.98 | 0.17 | 1.23 | 43.21 |

Temperature of water jacket, 24°C.

## EXPERIMENT 30

## Calorimetry. Specific Heat of a Metal

OBJECT: *To determine the specific heat of lead or copper.*

The metal in the form of shot (lead) or small discs (copper) is placed, together with a thermometer, in the inclined tube $S$

which passes through the boiler *B* in the apparatus shown in Fig. 25. It can thus be heated and at the same time kept perfectly dry. When the temperature as indicated by the thermometer *T* becomes steady (practically the boiling point of water), a slide *D* is raised and the shot allowed to fall into the calorimeter cup *C* which contains a known quantity of water. From the rise in temperature of this latter, the total heat contributed by the shot is found and its specific heat can be calculated.

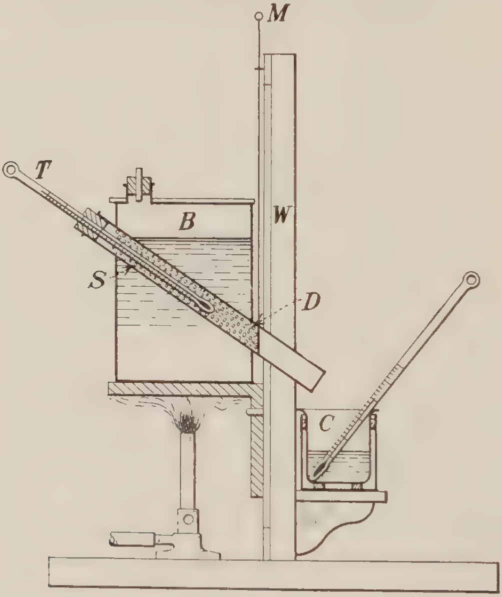

FIG. 25.

In performing the experiment, start with the boiler three-fourths full of water and be very careful not to let it boil dry. Weigh out carefully about 300 gm. of shot (or 150-gm. copper discs) and put it in the tube together with a thermometer. After the water has begun to boil and the temperature of the shot becomes constant place the calorimeter in position in its jacket. It should contain about 50 gm. of water weighed to a tenth of a gram (about 70 gm. if copper is used), cooled half a dozen degrees below room temperature. Now take the temperatures of the shot and the water and then allow the shot to fall into the calorimeter. Stir thoroughly with the thermometer and take readings every 15 sec. for 2 min. As it comes very quickly to its final temperature and then cools very slowly, cooling corrections may be omitted.

In many cases a somewhat simpler heating arrangement is used. This is entirely satisfactory if the shot is transferred *quickly* from heater to calorimeter.

Make two determinations of the specific heat, using slightly different amounts of *dry shot* for each run. In calculating the specific heat the student should build up his own equation; this is easier and more instructive than trying to find an equation which exactly fits this case. Decide which members furnish and which absorb heat. Remember that the calorimeter cup is heated as well as the contained water, so its water equivalent, or the amount of water which has the same heat capacity, must be figured and in the specific heat calculation this added to the mass of the water in the cup. The cup is made of copper and nickel-plated; neglect the container.

When using copper, a few of the discs sometimes stick in the heater tube; therefore always weigh the calorimeter and contents after the experiment is completed to determine the actual amount of copper used.

The following precautions must be observed to make the experiment successful:

1. Read all temperatures to a tenth of a degree and determine the temperature of the shot and of the calorimeter just before the shot is put in.
2. Begin to take readings of calorimeter temperatures as soon as the contents are thoroughly stirred. Take the first reading within 15 sec. after the shot has been transferred, if possible. Follow with a short series of readings at 15-sec. intervals. If these show a slight but fairly regular fall in temperature, it means that the calorimeter is cooling by radiation, convection, etc., and that the *highest reading* is the most nearly correct one to take.
3. In determining the masses of the quantities used, weigh the calorimeter first dry and empty, then with water, and finally with the water and shot.

### Questions

**1.** Derive the equation used in calculating specific heats from the data.

**2.** Why is it well to start with the calorimeter water somewhat colder than room temperature? If the total rise is 15°, how much colder than room temperature would it be well to have the water, assuming rate of increase as constant?

**3.** How is the water equivalent of the calorimeter computed?

**4.** What is the use of the protecting jacket around the calorimeter cup?

## EXPERIMENT 31

## Heat of Fusion

OBJECT: *To determine the heat of fusion of ice.*

When a solid is changed into a liquid state, heat is absorbed; when the reverse happens, heat is set free, and under similar conditions the same amount of energy is always transformed. In this experiment the latent heat of fusion of ice, that is, the energy required to melt 1 gm. of ice at its melting point, is to be determined.

The calorimeter is made of copper and nickel-plated to minimize radiation. It is futhermore protected from sudden and erratic changes in air temperature by placing it within a water jacket or double-walled vessel filled with water at room temperature. The calorimeter with its stirrer are to be weighed to a tenth of a gram on a platform balance and then filled with approximately 300 gm. of water at about 8° above the jacket temperature, the actual mass of the water being carefully determined. If it is necessary to heat the water, do not use the calorimeter, but heat the water in another vessel. Place the calorimeter within the water jacket; stir thoroughly and note the temperature; then add, without splashing, a piece of ice weighing about 60 gm., which has just been carefully dried. Now record at 1-min. intervals the temperature of the calorimeter, starting with the readings taken just before the ice was introduced. These readings are to be continued until at least five show a regular increase. Also read the temperature of the water jacket every 5 min. After completing the temperature readings the mass of ice used is determined by again weighing the calorimeter and contents.

Make two runs of this experiment and in the second case endeavor to have the water at such an initial temperature that it is as many degrees above the jacket temperature at the start as below it at the end of the experiment. This will minimize the errors due to radiation and convection. Compute the heat of fusion, first building up the equation. At the end of the experiment the calorimeter is to be emptied, but the water is to be left in the jacket.

If so directed by the instructor, follow the more accurate method of cooling correction described just before Exp. 30.

### Questions

1. Show how the equation used in this experiment was built up.
2. How much heat would be given out by 100 kg. of water in freezing?

## EXPERIMENT 32

## Vapor Pressure

Object: *To measure the saturated vapor pressure of a given substance for various temperatures.*

The pressure exerted by a saturated vapor depends upon its character and its temperature. In the present experiment the change in pressure of a saturated vapor with temperature will be studied. A special form of siphon barometer is used. In the space above the mercury, which would be a vacuum in an ordinary barometer, is the liquid whose vapor pressure is to be determined. The barometer is surrounded by a large glass tube filled with water which is connected with a heater tank by a pump and circulating system. It is assumed that the temperature of the vapor is the same as that of the water bath, but, owing to the slow conduction of heat through the glass walls of the barometer tube, the bath should be kept at constant temperature for several minutes before a reading is taken. To accomplish this, pump vigorously until the difference in readings between the upper and lower thermometer is less than 2°. By a little care in adjusting the flame under the heater, it will be possible to keep the temperature constant within 1° for at least 3 minutes and then it is safe to consider the temperature of the liquid and its vapor the same as that of the surrounding water.

By measuring the difference in height of the two mercury columns and taking into account the barometric reading, the pressure for any desired temperature may be determined. Start at room temperature and find the pressures for 10 temperatures varying between this and 85°.

**Curve.**—Temperatures as abscissas and pressures as ordinates.

### Questions

**1.** Explain on the molecular hypothesis how a vapor or a gas can exert a pressure.

**2.** Show by a diagram just how the vapor pressures were determined from the observations made.

**3.** What is meant by the boiling point of a liquid? How may it be determined from the observations for the substance used? Determine it, in this case, from the curve plotted.

**4.** What would be the physical state of a substance whose pressure and temperature are represented by a point *below* the curve? *Above* the curve?

## EXPERIMENT 33

## Relative Humidity

OBJECT: *To determine the relative humidity of the atmosphere under several different conditions.*

Atmospheric air, even in the dryest regions of the earth, always contains more or less moisture. While this is most in evidence when condensed in the form of fog or rain, it nevertheless makes itself felt when in the vapor form and it has long been known that its presence or absence has a great deal to do with bodily comfort.

Relative humidity is defined as the ratio of the actual amount of water vapor in a given volume of space divided by the maximum amount possible at this temperature, *i.e.*, the amount required to produce saturation. It is found that a relative humidity of less than 30 per cent is conducive to dryness of the skin and other and more serious discomforts, while a humidity above 80 per cent gives the air at summer temperatures a "muggy" feeling. A comfortable value for this climate is about 40 per cent (indoors) in winter and 70 per cent in summer.

## Part I

To determine humidity, the standard instrument is the wet and dry bulb hygrometer in the form of a "sling psychrometer." When this is whirled, the dry bulb takes the air temperature and the wet bulb is cooled by evaporation to a degree determined by the humidity present. *In whirling the psychrometer, keep well away from tables or other pieces of furniture.*

The readings are readily interpreted in terms of relative humidity with the aid of Table VIII at the end of this manual. For example, suppose the dry bulb read 21°C. and the wet bulb 14°. From the table it is seen that the actual vapor pressure (for $t = 21°$ and a difference of 7°) is 8.5 mm. of mercury while the saturated vapor pressure (no difference between wet and dry bulb) is (Table VII) 18.7 mm. Accordingly, since the amount of vapor is proportional to its pressure, the relative humidity is $8.5 \div 18.7 = 45.4$ per cent.

Make humidity measurements: (1) in a laboratory where many students are working, (2) in an empty room, and (3) out of doors (if not too cold).

## Part II

Check the relative humidity measurements (made in the laboratory) of Part I by actual determination of the *dew point* or temperature of condensation. To do this, take one of the small calorimeter cups—choose one with a well-polished bright nickel surface—and fill it about one-third full of water. In this dissolve, a little at a time, ammonium nitrate crystals, stirring with a thermometer. It will be noted that the temperature at once begins to fall, because ammonium nitrate has a negative heat of solution (*i.e.*, this is an "endothermic" or heat-absorbing reaction), and before long the surface of the cup will be clouded with a film of moisture, or dew.

Observe carefully the temperature at which this film just begins to form, then add a little water to raise the temperature, noting this when the film disappears. Again, add a little more nitrate so as to cool very slowly down to the dew point and allow a careful final determination. Take the mean of the temperatures at which the film appears and disappears as the dew point. Calculate the relative humidity with the aid of Table VII.

When through, pour the solution into the jar provided for this purpose, so that the nitrate may be regained by evaporation. *Rinse out the calorimeter cup.* Another way of cooling is by the use of crushed ice (with salt, if necessary) in the water.

**Precautions:**

Keep as far away from the cup as convenient and be particularly careful not to breathe in its direction, as the moisture of the breath or the body will affect the result. It is probably better for one student to do the stirring and reading of temperatures, while the other watches the surface. In very accurate work, this is done from a distance of several feet, with a telescope. Avoid a strong draft of air in this part of the experiment.

### Questions

**1.** From the tables mentioned above, compute the absolute humidity or number of grams of water per cubic meter of space, in each of the above cases. To do this, find the amount necessary for saturation and then make use of the relative humidity determinations. Make a rough computation of the total amount of water present in the air in the laboratory.

**2.** Is it correct to say that a hot-air furnace *dries* the air in the sense of lowering (appreciably) its absolute humidity? Does it lower its relative humidity?

**3.** How would one go about it to raise the humidity in a house heated by hot air? How much water would have to be evaporated to raise the relative humidity of a house of 500 cu. m. from 20 to 50 per cent, the temperature being 22°C.?

**4.** Explain in detail how the method of Part II gives the relative humidity.

**5.** Explain why cold-water pipes and other cold surfaces are more likely to "sweat"—*i.e.*, show condensation—in summer than in winter.

## EXPERIMENT 34

### Melting Point of a Substance. Cooling Curves

OBJECT: *To determine the melting points of various substances.*

If a substance is hotter than its surroundings, its temperature, in general, will fall, since it is constantly losing heat through radiation and convection. If readings of the temperature of the body are taken at definite intervals of time and the results plotted, the curve thus obtained shows this change, and is called the cooling curve of the substance. From it many important thermal properties of the body can be determined. Such a curve will be used in this experiment to find the melting point or point of solidification of a substance, for, when a liquid is changing into the solid state, heat is set free which compensates for that lost to the neighboring bodies. Consequently, for a time its temperature ceases to fall, and this constant temperature is the melting point of the material.

In the actual experiment cooling curves for three substances will be determined, one of which is water. This is included to show the form of curve for a body which does not pass through a change of state, that is, in the temperature interval involved in this experiment. The materials are contained in short test tubes attached to a vertical rod; inserted in these tubes are thermometers. In order to raise the temperature of the substances, a water bath is also attached to the rod and the position of this bath can be changed. To start the experiment, the bath is raised until the substances contained in the tubes are completely immersed; a burner should never be brought into direct contact with the test tube.

When the temperatures of the substances are between 85 and 90°, the burner is to be removed and the water bath lowered and turned so that the vapor will not rise and come in contact with the tubes. Readings of the temperatures of all the tubes are then to be taken over an interval of 40 min. During the first 15 min., readings every half minute are to be recorded, from 15 to 25 min., readings every minute, and from 25 to 40, every 5 min. During this time the substances are not to be stirred or jarred in any way, for the success of the experiment depends largely upon the apparatus being kept perfectly quiet. Hence the thermometers should be read without touching or twisting them. Also a place in the laboratory should be chosen where the apparatus will not be subjected to drafts.

**Curve.**—All the data are to be included on one sheet. Times as abscissas and temperatures as ordinates.

### Questions

**1.** Determine melting points of materials used.

**2.** Define melting point, heat of fusion, supercooling.

**3.** Interpret the curves, paying special attention to the following points:

(*a*) When the substances are all liquids, do they cool at the same rate? If not, why not?

(*b*) If the temperature remains stationary or rises, it means that heat is supplied the body. Where does it come from?

**4.** If equal masses of two substances having the same melting points but different heats of fusion are placed in the apparatus and cooling curves taken, would the curves be the same? Indicate the difference by sketches.

## EXPERIMENT 35

## The Mechanical Equivalent of Heat

OBJECT: *To determine the mechanical equivalent of heat.*

The law of the Conservation of Energy says that when energy disappears in one form it must appear in another. Thus, when a falling body strikes the ground, its energy may be partly spent in disrupting the body and partly—a very small portion, indeed—in producing a sound; but the largest share of it is converted into energy of molecular motion, that is, *heat*.

In this experiment the heat produced in this way, which results from the expenditure of a definite amount of work, will be measured. A quantity of buckshot will be allowed to fall many times from end to end of a long wooden tube. The mechanical energy expended is computed from the total length of fall of the shot while the heat energy gained is measured by noting its rise in temperature. The result will be expressed as the number of ergs of work which are required to produce a calorie of heat, *i.e.*, the mechanical equivalent of heat.

Empty the shot into the tin pan and cool it about 5° below room temperature. In winter this may be done by placing it outside a window; at other times it can be put on a block of ice. *The shot must always be kept dry.* Then pour it back into the asbestos-lined box and measure its temperature carefully, taking several readings to a tenth of a degree. Pour it into the tube and stop the end; then invert the tube 100 times, being careful neither to raise nor to lower it in so doing; it must be done quickly enough so that the shot falls the entire length of the tube each time. After the hundredth inversion pour out the shot and record its temperature. From the specific heat of lead (0.0315) and the rise in temperature, the heat developed per gram is obtained, and from the length of the fall the expended energy is found. The mechanical equivalent is then readily calculated. Make three complete determinations.

The following precautions must be observed:

1. Do not trust to a *single* thermometer reading at either the beginning or the end of the experiment. Make three in quick succession, shaking up the shot each time.

2. Remember that each shot does not fall the entire distance from end to end of tube. The *average* distance will be from the top of the pile at one end to the bottom at the other.

3. Invert the tube by holding it in the center and be careful not to raise or lower it in so doing—this can be done most conveniently while seated in a chair. Have the end of the tube on the floor when the shot falls.

### Questions

**1.** Is it necessary to know the weight of shot used in this experiment? Why?

**2.** Why must the shot be cooled below room temperature at the start?

**3.** Calculate, using the result calculated for the mechanical equivalent, how much the water is warmed by passing over Niagara Falls and falling 50 m.

**4.** Would the vigorous stirring of the water in a calorimeter introduce an error in highly accurate work?

**5.** What are the chief sources of error in this experiment? Could a brass tube be used?

## EXPERIMENT 36

## Mechanical Equivalent of Heat. Callendar's Apparatus

Object: *To make a careful determination of the mechanical equivalent of heat, using Callendar's apparatus.*

By the mechanical equivalent of heat is meant the ratio between the mechanical and the heat units of work. This is one of the most important of the physical constants and has been determined by many methods, most of which are long and tedious. The apparatus designed by Professor Callendar, however, enables one to make a fairly good determination in a short time, though for accuracy and reliability it cannot approach the more intricate forms. The following description of the apparatus and the method of performing the experiment is taken from a leaflet published by the Cambridge Scientific Instrument Company.

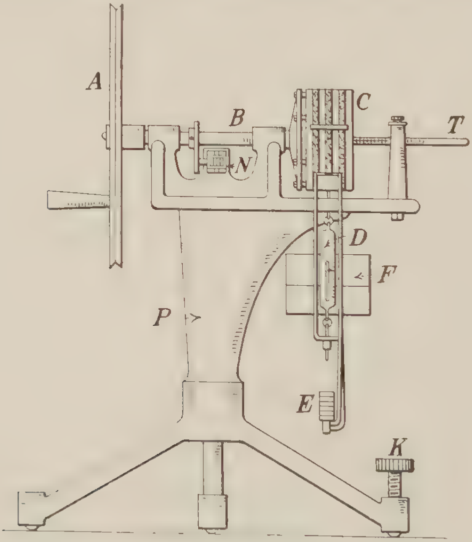

Fig. 26.

This apparatus has been designed by Prof. H. L. Callendar, F. R. S., to provide a rapid and ready means of determining $J$, the mechanical equivalent of heat. In it mechanical energy is dissipated by means of a special brake rubbing on the outside of a rotating brass drum. The heat energy developed is measured by the rise in temperature of the brass drum and the water it contains.

Referring to Fig. 26, $C$ is the calorimeter drum made of thin brass and suitably mounted for rotation about the horizontal axis $B$. The driving wheel $A$ may either be turned by hand or driven by an electric motor. The number of revolutions made by the calorimeter drum is automatically recorded by the counter $N$. The brake consists of a silk belt arranged to form one and one-half complete turns around the calorimeter drum. Unequal adjustable weights $E$ and $F$ are suspended from the ends of this belt, and stability of equilibrium is secured by the addition of a light spring balance acting in direct opposition to the lighter weight. The extreme flexibility of the belt insures that, to a very high degree of approximation, the difference of load at the two ends is a true measure of the friction.

The rise in temperature of the calorimeter $C$ and the water it contains is observed by means of a mercury thermometer $T$, inserted through an axial opening in the front of the cylinder. The bulb of this thermometer is bent round so that it is fully immersed in the water.

The motion of the surface of the calorimeter reduces the effect of draughts and convection currents, so that the loss of heat is much more regular than if the surface were at rest.

*Instructions for Use.*—The surface of the brass drum should be kept as smooth and bright as possible. Before making an experiment, it is best to polish it lightly with a clean duster and a little French chalk. The silk belt should also be kept clean and dry, being wrapped up in clean paper and put away when the apparatus is not in use. When in use, the belt is wrapped around the calorimeter so as to encircle it one and a half times; three-quarters of the circumference of the calorimeter is thus overlaid by the single part of the belt and a like amount by the double part. From the single part is suspended a brass frame which carries the weight $E$ and which is also connected to the spring balance $D$. From the double part of the belt the heavy weights $F$ are suspended. After the belt is correctly mounted in position, a weighed quantity of water, say about 350 gm., should be introduced by means of a pipette, into drum $C$. Care must be taken not to allow any water to fall on the belt or surface of the drum, or the friction will not be steady. The water used should be at a temperature of 5 or 6°C. below the atmospheric temperature. The special bent thermometer $T$ can then be fixed in position with its bulb immersed in the water in the drum.

For the heavy weight it is usually convenient to use about 4 kilos. On starting to rotate the calorimeter, the small weights on the opposite side should be adjusted until the 4-kilo weight is lifted clear of its stop and held in floating equilibrium with the pointer of the spring balance somewhere near the center of its scale. The speed of the calorimeter drum should be from 60 to 120 r. p. m.

After a few revolutions the friction will become practically constant, and the temperature will rise at the rate of about 1°C. per 100 revolutions. The reading of the thermometer $T$ should be observed every 100 revolutions. The mean temperature of the air surrounding the calorimeter should also be taken.

Select from the observations a range of 500 or 600 revolutions during which the mean temperature of the calorimeter is nearly the same as that of the air. The corrections for external radiation will then be practically negligible.

The work done is the product of the difference of the weights on the two sides (the heavy weight plus the spring balance reading minus the light weight) by the number of revolutions and the circumference of the calorimeter, which latter should be measured with a thin steel tape. In reckoning the total weights on either side, the metal parts which carry the loose weights should be included. The heat generated is the product of the total thermal capacity of the calorimeter and its contents by the observed rise of temperature. The weight of the calorimeter drum is stamped on it and, as the specific heat of the brass used is about 0.092, the water equivalent of the calorimeter can be readily calculated.

While the apparatus as used in this laboratory does not correspond in every respect to the above description, these instructions will nevertheless serve very well. Two students can perform the experiment, but three and even four are not too many to work at it at one time. One student turns the handle and an alternate relieves him (the calorimeter must be rotated *continuously* at about 100 r. p. m. throughout the duration of a run, which lasts, say, 10 min.). Another student watches the counter and calls each 50 revolutions, while a fourth records the thermometer readings at these times. The spring balance may be read between times. From these data plot a curve of revolutions against temperatures and draw a tangent to this curve at room temperature. From the slope of this tangent and the other data the mechanical equivalent may be determined.

Make two complete determinations.

**Curve.**—Revolutions as abscissas and temperatures as ordinates.

### Questions

**1.** Explain in detail how the mechanical equivalent is computed from the data obtained. In what units is it expressed?

**2.** How does the above method eliminate radiation correction?

## EXPERIMENT 37

## Thermal Conductivity of Metals

Object: *To determine the coefficients of heat conduction, or thermal conductivities, of copper or iron, in c. g. s. units.*

The equation of heat conduction,

$$Q = K\frac{At\,(T_1 - T_2)}{L},$$

expresses the fact that the quantity of heat $Q$, which flows by conduction along a rod is proportional to the area of cross-section $A$ of the rod, and to the difference of the temperatures, $T_1$ and $T_2$, at its two ends; also to the time (expressed in seconds). It is inversely proportional to the length $L$. The constant of proportionality $K$ in the above equation is called the *thermal conductivity* of the substance under test. In determining $K$ the amount of heat which flows by conduction in a given time along a rod of copper or iron, of known length and cross-section and with a known temperature difference between its ends will be measured with a calorimeter.

The copper pan $P$ (Fig. 27) is to be filled three-fourths full of water which is kept boiling vigorously. The water level must always be maintained well up towards the top during the course of the experiment by occasionally adding hot water or cold water in such small amounts as not to stop the boiling. The temperature of the upper end of the rod is determined by a thermometer stuck in the hole in the copper block $E$, into which is fastened the end of the rod $R$, the conductivity of which is to be determined. The lower end of the rod extends into a calorimeter containing about 75 gm. of water, weighed carefully. This must be adjusted at such a height that the water level comes just above the copper disc, but care must be taken that the cotton wrapping on the rod does not get wet. The calorimeter temperature is read by a thermometer running through the guide on the copper pan and through the hole in the disc. It is best to use for this purpose one of the 0 to 60° thermometers.

Determine room temperature in the neighborhood of the apparatus and start with water cooled at least 8° below this temperature. After the thermometer $V$ shows the upper end of the rod to have reached a steady temperature—usually about 96°—put the calorimeter in place and make a series of readings of calorimeter temperatures every half minute until the water is heated 5° above room temperature. In making these readings do not strain the eye by watching the thermometer continually, but get ready for a reading 5 sec. before the time. Also read the thermometer at the upper end of the rod every 2 min. *The calorimeter must be stirred continuously during the whole series of readings.*

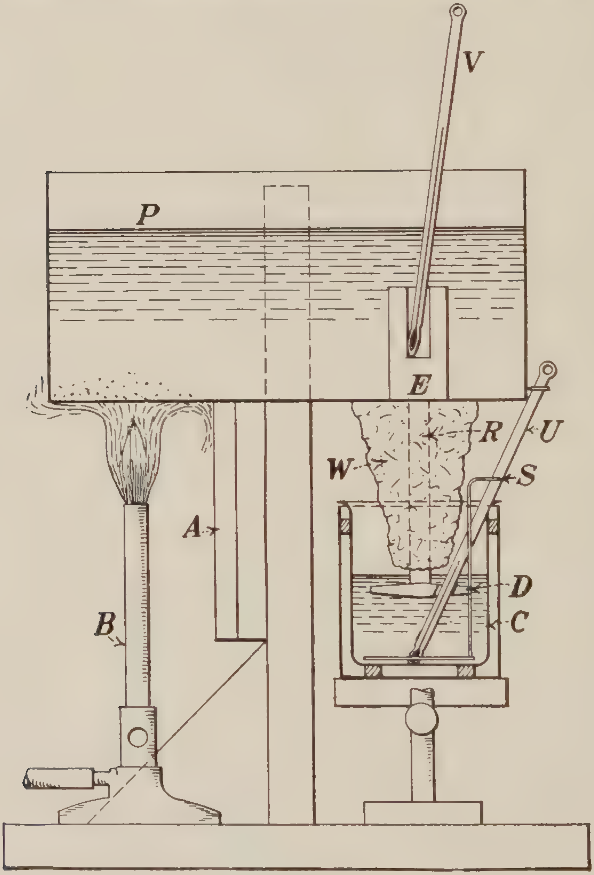

FIG. 27.

The best method of reducing the observation is as follows: First rewrite the conduction equation in the form

$$K = \frac{L}{A(T_1 - T_2)} \cdot \frac{Q}{t}.$$

This expresses the fact that the conductivity is proportional to $\frac{Q}{t}$ or the *rate of gain of heat* by the calorimeter. To determine this, plot a curve with times as abscissas and calorimeter temperatures as ordinates. With a ruler draw a tangent to this curve at room temperature. This gives the rate of rise *when the calorimeter is neither gaining nor losing heat by radiation*, in other words, just what is wanted. Now $\frac{Q}{t}$ is not the rate of temperature rise of the calorimeter, but its *rate of gain of heat*. Knowing the amount of water which is being heated and its rate of temperature rise, this latter is readily calculated. Do not neglect to take account of the water equivalent of the calorimeter cup, stirrer, and (25 gm.) copper disc. This may be considered as all copper (specific heat 0.092). For accurate work it would be well to estimate the water equivalent of the thermometer, considering the part heated to consist chiefly of mercury.

Of the other terms, $L$ and $A$ are given for each piece of apparatus, but must be expressed in terms of the centimeter as a unit. $T_1$ is the average temperature of the hot end of the rod, while $T_2$ is the temperature of the cold end just at the time $\frac{Q}{t}$ is determined; that is, it is room temperature.

Make two determinations of thermal conductivity. The instructor will specify whether these should both be for the same material, or one for copper and the other for iron.

*Please leave the stirrer in the copper pan when the experiment is completed.*

**Curve.**—Times as abscissas and calorimeter temperatures as ordinates.

### Questions

**1.** From the results, which material, *i.e.*, copper or iron, would be best for a poker? A cooking vessel?

**2.** Calculate the heat which would escape in an hour from a window of 1-sq. m. area made of glass of 4-mm. thickness with a temperature difference of 2°C. between its faces. Conductivity of glass is 0.0024.

**3.** Why must the calorimeter be stirred constantly during the experiment?

**4.** What sources of error can be mentioned in this experiment?

## EXPERIMENT 38

## Thermal Conductivity of Heat-insulating Materials

OBJECT: *To measure the thermal conductivity of several poor conductors by comparison with hard rubber.*

(Note.—This experiment is suitable for either two or four students.)

If two flat plates are in contact, face to face, and a steady flow of heat is established through them—perpendicular to the broad faces—the heat $Q$ which passes in time $t$ is given by

$$Q = k_1 \frac{D_1 A t}{l_1} = k_2 \frac{D_2 A t}{l_2},$$

where $k_1$ and $k_2$ are the conductivities, $l_1$ and $l_2$ the thicknesses, and $D_1$ and $D_2$ the temperature drops through the plates 1 and 2 respectively. Since the area $A$ and time $t$ are the same in each case,

$$\frac{k_1}{k_2} = \frac{D_2}{D_1} \cdot \frac{l_1}{l_2}.$$

In other words, the ratio of conductivities is given by the ratio of temperature differences and thicknesses. The actual quantity $Q$ of heat flowing need not be measured.

This is the principle of this method of determining thermal conductivity, which was originally due to Christiansen. The comparison substance in this case is hard rubber or vulcanite, the conductivity of which may be taken as 0.00037 c. g. s. units. This vulcanite plate $V$ (Fig. 28) and the specimen $S$ to be tested are placed together between a hot and a cold surface. Measurements of the temperatures of these two and of the boundary face between the vulcanite and the specimen give the two temperature differences $D_1$ and $D_2$.

*The apparatus* includes an electric heater, a copper boiler, and a large containing box which is to be unpacked and the contents checked with the following list:

1. Four brass cells for steam or cold water.
2. One 9-in. and nine 3-in. squares of vulcanite.
3. Sixteen brass plates (handle carefully to avoid nicking faces).
4. Several specimens of commercial wall insulators and one of wood—in nine small squares.
5. A faucet hose connection, rubber tubing, cotton batting, and sheet asbestos.

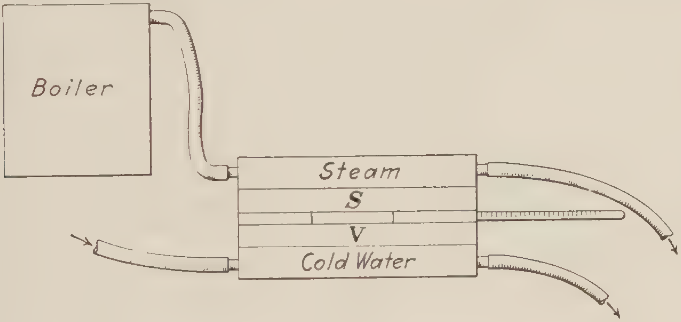

Fig. 28.

Two special thermometers (0 to 100°C.) will be furnished by the instructor.

Two simultaneous tests of fair accuracy may be carried out (Part I) or one test of greater accuracy (Part II). As it requires at least 6 hr. to reach a steady temperature state, the apparatus should be started in the morning and the final temperatures read in the afternoon, or else started in the afternoon and the readings taken the next morning. The boiler will run nearly 24 hr. with one filling

## Part I

Fill the boiler to within an inch of the top with distilled water (Room 31 in basement) and start it heating. Examine the four brass cells and note that each has one *flat* face (the cast brass, *not* the sheet brass side). Two of these are to be cold cells and carry circulating tap water—lying flat side up in the bottom of the box—while two are connected with the boiler by a short tube. There are channels inside each cell, so that the water or steam must circulate to every part.

Connect the two cold cells with the tap so that water circulates through them in series and returns to the sink. Place on each a vulcanite plate (made up in one case of the small blocks placed close together) to form the comparison plate, and on this place the brass plates in a compact square but in such a way that there is left a little channel through which the thermometer can extend into the center. The sole purpose of these brass "thermometer" plates is to provide a means of getting the temperature of the vulcanite and specimen surfaces with the aid of a mercury thermometer, thus avoiding the use of a thermocouple. Since the conductivity of brass is several hundred times greater than that of hard rubber, the effect of these plates on the heat flow is negligible.

Then put in place the two specimens to be tested—perhaps wood (pine) and some commercial insulator for the first trial. If the specimens are less than a centimeter thick, use two or more thicknesses. Put the steam cells in position with the flat side *down* (be sure the *cold* cells are flat side *up*), connect the steam line, and run the return to the sink. Put a sheet of asbestos on the top of the hot cell and, except for easily compressible substances, load with several kilogram-weights. Pack the cotton wool carefully around the edges, but do not leave any outside the box. Put the thermometers in place and put a little loose cotton in the slits in the box. Close, and fasten the lid. Protect the steam pipe between the boiler and the box by surrounding it with a little asbestos tube, which is easily made up. See that the water is running a good stream—about three liters a minute—and that there are no kinks in either steam or water line. Note particularly the special instructions at the end of Part II.

If possible, arrange to return after a few hours to see that water and steam are flowing well and that everything else is going as it should. No usable temperature readings can be taken, however, until steam has passed for several hours. Then read the thermometers carefully. Also measure the tap-water temperature (average of inflow and outflow) and get the steam temperature from the barometric reading. If such arrangements have been made with the instructor, refill the boiler and put two other specimens under test. Calculate the conductivities.

### Part II

Having gained an acquaintance with the experiment, more accurate results can now be obtained with a slightly different arrangement. Only one specimen will be tested at a time, and this is to be chosen with some care. If the material is soft and easily compressible (blankets, balsam wool, etc.), build up outside the box and determine the thickness of the specimen under compression. In some cases it is desirable to insert little fiber plugs to hold the plates apart and to prevent the sample from being compressed.

Use only one hot and one cold cell. Put the nine small vulcanite plates in a compact square on the cold cell—they are in small squares so as to make better contact but they must *fit closely together*. It will be found that they vary slightly in thickness, but by taking a little pains they can be put together in such a way as to make a fairly even surface. On these place the brass thermometer plates, then the specimen, then the other series of thermometer plates, followed by the large vulcanite plate, and, finally, the steam cell. In this way the temperature drop through the vulcanite *on each side* of the specimen is measured and the average of these compared with the drop through the specimen. See that all the small plates fit compactly together, put the two thermometers in place, and have steam and water flowing at least 6 hr. before making final temperature readings.

It is a good plan to place a block 2 or 3 in. thick under the end of the box next the boiler and to have the water enter at the opposite end. This tends to prevent water collecting in the steam cells, and air in the water cells.

At the end of the run, read each thermometer several times and use the average value. Find steam and water temperatures as before and determine zero errors of the thermometers by inserting in cracked ice. In calculating the conductivity of the specimen, it is best to subtract 1°C. from the temperature drops through the specimen and through the vulcanite, to allow for the effect of the several surfaces.

**Special Instructions** (for either part of experiment):

1. Use only good sound hose for the water line. A water leak overnight might be a serious affair. Hose used with steam soon becomes soft and flabby. Unless this is in too bad shape it may still be used on the steam line, but not on the water line. In all cases use as short lengths of hose as convenient. Remove from the sink any paper or any other debris which might cause it to clog.
2. Avoid fire danger. See that no loose cotton is left near the heater or, for that matter, anywhere outside the box. Close and fasten the lid of the box before leaving. The electric heater should be on two thicknesses of asbestos.
3. The student should thumb-tack a note on the cover with his name and the time of starting the experiment (day and hour). He should not neglect to return and take the readings at the proper time. If the apparatus is neglected and the boiler goes dry, not only will the experiment be spoiled but the instructor will be at liberty to turn the apparatus over to another group.
4. Some difficulty will be experienced in getting the proper water flow because ordinary faucets, when running a small stream, tend to shut off. It is best to start the water flow at four liters a minute (actually measured) and then, return after an hour and adjust it to about three. It should not, at any time fall below one and one-half liters a minute. Before leaving apparatus make a careful inspection for leaks.
5. At the conclusion of these tests leave the apparatus just as it is. Turn off the stove and water and return thermometers to the instructor. Do not empty the boiler.

### Questions

**1.** The principal source of error is probably the unavoidable escape of heat from the sides, in spite of the cotton packing. Would the use of plates of larger area lessen this error?

**2.** The effect of this loss is to cause more heat to pass through the specimen as tested in Part I than through the vulcanite plate. Would this cause the measured conductivity to be too high or too low?

**3.** Why has the central thermometer plate little nicks on the edges to prevent it from coming in contact with the others?

**4.** Calculate the heat loss through 100 sq. m. of the wall insulator tested, in a day. How much coal (heat of combustion 8000 cal./gm.) would be saved by using two thicknesses? Assume a 10°C. temperature drop through the insulator in each case.

**5.** (Part II only.) Show how this method eliminates the error of 1.

## EXPERIMENT 39

### Isotherms

OBJECT: *To determine the relation between pressure and volume of a gas at two temperatures.*

The physical state of a substance is specified when it is known under what pressure and temperature it exists and what its volume is. The simplest way of showing these relations is by curves which have for their coordinates two of these variables while the third remains constant. Thus, three sets may be obtained, but in this experiment only one, the isotherm, will be determined. The apparatus consists of an open and a closed glass tube placed parallel to each other but far enough apart so that a metric scale may be placed between them. They are connected by a rubber tube filled with mercury which traps air in the closed tube. Variations in the volume and pressure are obtained by placing the movable tube in different positions, thus causing the mercury levels in the two columns to change. The volume is obtained from graduations on the tube. *Read the barometer.* The pressure is determined in centimeters of mercury and is found by taking the algebraic sum of the atmospheric pressure and the difference in the mercury levels. Take six readings, using pressures both above and below atmospheric and covering as wide a range as possible. Do this for two temperatures, that of the room and also that of boiling water; be sure to use the same apparatus for each run. The higher temperature is obtained by sending steam through the jacket which surrounds the closed tube. Determine the product of the pressures and the corresponding volumes, and find from the average of these products the value of $R'$ for each run, $R'$ being defined by the equation

$$PV = R'T,$$

where $T$ is the absolute temperature.

In doing the arithmetic work, carry the numbers representing pressures and volumes to the three significant figures and the products to four. The table of reciprocals found at the end of the book will help in computations.

**Curves.**—

Sheet 1

Volumes as abscissas and pressures as ordinates, the data for both temperatures being included.

Sheet 2

Volumes as abscissas and reciprocals of pressures as ordinates.

### Questions

**1.** Interpret the curves.

**2.** Explain how the coefficients of expansion of the gas could be determined from the curves.

# PART IV

# ELECTRICITY AND MAGNETISM

## EXPERIMENT 40

## Faraday's Ice-pail Experiment

Object: *To study the phenomena of electrostatic induction as illustrated by Faraday's ice-pail experiment.*

This classical experiment illustrates the fundamental phenomena of electrostatic induction. On account of occasional irregularities the observations should be repeated at least twice to make certain that the apparatus is working as it should.

Certain precautions must be observed if the experiment is to be successful.

Handle the stand, which is to support the copper vessel, only by its base, so as not to lessen its insulating properties.

2. Make certain that the wire connecting the electroscope and the conductor does not touch the outside of the electroscope, or any other object.

3. Always keep the discharge terminals of the static machine separated so that a spark cannot pass, for that may reverse the polarity of the machine. Always bring the test ball in contact with the same terminal of the machine.

4. Be careful in bringing a charged body near the electroscope because the instrument is very sensitive and is easily injured, especially if the charged body is brought near the glass plates. Keep the electroscope at some distance from the static machine or incorrect results will be obtained.

5. It should be noted that the leaves of an electroscope do not always come completely together even when the instrument is discharged, due to the rigidity of the leaves.

6. Ground the case of the electroscope by attaching it to a gas main or a water pipe.

Make the following observations, paying particular attention to the effect of each operation on the electroscope.

1. Charge the electroscope by placing the test ball in contact with one knob of the static machine while it is running and then touching it to the knob of the electroscope. Test the sign of the charge on the electroscope by bringing near it a hard-rubber rod which has been given a negative charge. Discharge the electroscope.

2. Connect the electroscope with the conductor, which is in this experiment a copper vessel, *i.e.*, the "ice pail" on the insulating stand. Recharge the test ball from the same terminal of the machine and lower it into the vessel, but without making contact. Test the sign of charge on the electroscope. Remove the test ball. During the entire operation the ball must not touch the conductor.

3. Recharge the test ball and again lower into the conductor without contact. Now ground the conductor by touching it with the hand. Break the ground and remove the test ball. Test the sign of charge on the electroscope. Discharge the electroscope.

4. Recharge the test ball and lower it into the conductor, allowing it to touch the latter. Remove the test ball. Test sign of charge on the electroscope. Discharge the electroscope.

5. Recharge the test ball and lower it into the conductor without contact. Ground the conductor. Break the ground and allow the test ball to touch the vessel. Remove the test ball.

The report should consist largely of *diagrams* which show the distribution of charge on pail, ball, and electroscope at each step of the operation. *Under each diagram* explain briefly why the charges are so distributed.

### Questions

**1.** What fundamental facts of electrostatic induction have been demonstrated by this experiment?

**2.** Without touching the electroscope, how could it be determined whether it is discharged or slightly negatively charged?

**3.** In grounding the conductor (operation 3), does it make any difference whether the outside or the inside is touched with the hand? Justify the answer.

**4.** Define potential and describe the variation of potential of the vessel during operation 3.

## EXPERIMENT 41

## The Gold-leaf Electroscope

Object: *To study the electroscope as a potential indicator; in particular, to compare the charge and also the potential on the inside and the outside of a hollow conductor.*

The gold-leaf electroscope is usually looked upon as an instrument for the detection of an electric charge and for the determination of its sign. It can also be used to indicate the potential of a body, and the present experiment has been planned to emphasize this use of the instrument. Except in operation 2, the case of the electroscope is to be grounded.

1. Charge the electroscope by induction. To do this, bring near the knob a hard-rubber rod, charged negatively. Ground the electroscope while the rod is in its immediate vicinity. Break the ground and then remove the rubber rod. During these operations, note particularly the movement of the leaves and discuss these movements with respect to the total charge on the electroscope and also in terms of potential.

2. Place the electroscope on an insulating stand and connect the case of the instrument with its knob by a wire. Bring a charged body near the electroscope. Do the leaves move? See if the electroscope is charged by using a proof ball and another electroscope. Explain these results.

3. Charge an insulated spherical conductor by contact and investigate the distribution of charge on the inside and the outside of the sphere, using a proof ball and electroscope. Then attach the proof ball to the uncharged electroscope by a fine wire and note the deflection of the instrument as the proof ball is moved over both the inner and the outer surfaces of the sphere. Explain the differences in the action of the electroscope when the two methods are employed to study the charge on the sphere.

### Questions

**1.** How may an electroscope be used to measure potential?

**2.** A small proof ball is touched first to the outside and then to the inside of a charged spherical conductor. Does it acquire the same charge in the two cases? The same potential?

## EXPERIMENT 42

## Electrical Condensers

OBJECT: *To study the principle of the condenser; in particular, to determine the effect on the potential of a charged conductor of bringing near it a charge of opposite sign.*

Condensers are made in a variety of forms to meet different requirements of service, but they may be divided into two general classes; those of small capacity intended for use with high differences of potential, and those designed for low potential differences, which can be made with very large capacity. In this experiment two examples of the first class will be used, the plate condenser and the Leyden jar. The former is selected because it illustrates so well the general theory of the condenser, and the observations outlined are intended to show the change in potential of a charged insulated conductor when other bodies are brought in the neighborhood.

### The Plate Condenser

As the plate condenser has small capacity, the quantity of electricity which can be stored on it is not large, so that special care must be taken to prevent leakage of the charge. Also the electroscopes have certain peculiarities which often lead to incorrect results. Hence the following precautions must be observed in performing the experiment:

1. Place the apparatus and the electroscope far enough away from the static machine so that the strong charges on the latter will not affect the instruments.

2. Before starting the experiment, discharge the electroscope by touching the case and the insulated knob at the same time. The leaves do not always come together even when the instrument is discharged because of the rigidity of the foil. This is especially true of the electroscope provided with aluminum leaves. Connect the outer case of the electroscope to a gas main or water pipe.

3. In handling the condenser, do not touch the rubber blocks which support the plates, for this lessens their insulating properties.

Write at the top of the data sheet, before beginning the experiment, the definitions of *potential* and *capacity*. Then perform the following operations, explaining briefly on the data sheet the reason for the effects obtained. Interpret all movements of electroscope leaves in terms of the change in potential of the conductor to which the electroscope is attached, *i.e.*, the fixed plate in all of the following cases. The movable plate in every case is grounded. Use as a (nearly) constant source of charge a Leyden jar charged from the static machine by connecting the outer and the inner coats to the two terminals. Handle the jar by the outer coating and be careful not to touch the knob.

**1. Effect of Plate Separation on Capacity.**—*a.* With plates widely separated, charge the fixed plate sufficiently to give a divergence of the electroscope leaves of about 1 cm. (One or two charges from the test ball should effect this.) Now bring the movable plate very close to the other and observe the potential (measured by leaf divergence) which is now produced by the same charge; also how many charges will now be required to produce the same potential as before? Why is the potential of the fixed plate changed by the approach of a grounded plate? How is the capacity of the fixed plate changed during this operation?

*b.* With plates as nearly parallel as possible and placed successively 4, 3, 2, 1 mm. apart, determine the number of charges required in each case to produce approximately the same potential.

**2. Need of Grounding One Plate of Condenser.**—Repeat operations of 1*a* with the movable plate insulated. Why does the movable plate affect the potential and capacity appreciably only when it is grounded?

**3. Effect of Dielectric.**—Separate the plates so that a plate of glass can just be inserted, and compare the number of charges required to give a definite potential to the fixed plate, first, with air for the dielectric and, second, with the glass plate inserted. What effect does the glass have on the capacity?

### Questions

**1.** Since the electroscope really measures the potential of the fixed plate, try to determine the law connecting this potential with the distance between the plates, for a fixed charge.

**2.** The *capacity* of a condenser is defined as the amount of electrical charge required to raise its potential by unit amount. How has the capacity of this plate condenser been found to vary with the distance between plates?

**3.** What is the effect on the potential of the condenser of introducing the sheet of glass? On its *capacity?*

**4.** If one plate of a condenser of capacity of 12 units were grounded and the other charged to a potential of 8 units, what would be the magnitude and sign of the charge on each plate? What would be the "charge on the condenser"?

## The Leyden Jar

The Leyden jar can hold a sufficient quantity of electricity to cause very disagreeable sensations if it is accidentally discharged through a person's body. Hence care should be exercised in handling a jar. The knob or any part connected with the inner coating should never be touched unless the jar is on an insulating stand and even then care must be taken not to touch the outer and the inner coatings at the same time. The following observations are to be made:

4. Place the jar on an insulating stand and connect the inner coat with one terminal of the static machine, the other coat being insulated. Charge the jar. Test the signs of the charges on the inner and the outer coats, using a proof ball and an electroscope.

5. Discharge the jar, using the discharge rod provided with an insulated handle. Note the length and the brightness of the spark.

6. Connect the outer coat of the jar to the static machine, having the inner one connected to the other terminal. Charge the jar and determine the signs of charges present. Discharge the jar, noting the character of the spark.

7. Charge the jar and disconnect both coats from the machine. Discharge the jar. Wait 5 min. and again discharge it.

### Questions

**5.** Explain each of the observations in detail, *i.e.*, explain why the leaves of the electroscope move as they do, not simply state that they move.

**6.** Explain briefly the action of a condenser, either plate or Leyden jar.

**7.** What is residual charge?

## EXPERIMENT 43

## Magnetic Fields

### Part I

Object: *To map the resultant field due to the magnetism of the earth and a permanent bar magnet.*

Fasten with thumb tacks a large sheet of plain white paper to the board furnished for this experiment. Upon this, near the center, place a bar magnet with its axis in the direction specified by the instructor. Outline the magnet on the paper; indicate its polarity and *the direction of the earth's field.* Place the small compass near the north pole of the magnet and make dots as near each end of the needle as possible, and in line with it. Then move the compass in the direction in which its north pole points until the south pole is above the dot previously made at the north pole and make another dot at the north pole in its new position. Continue until the series of dots leads to the south pole of the bar magnet or near the edge of the paper. Draw a smooth curve through the points and indicate, by arrows, the direction in which it was traced. In a similar way trace other lines of force until the field is clearly represented on all sides, continuing until lines have been traced far enough from the magnet to show the undisturbed field of the earth. Of course, it is not necessary to start in each case from the north pole; it is just as well to start from the south, or, for that matter, from some point away from the magnet.

Two places will be found, where the direction taken by the compass needle is indeterminate, *i.e.*, points of neutral equilibrium. These positions are called *neutral points*, and the region in their immediate neighborhood should be mapped with particular care. The field near each of these points is very weak and is not always able to overcome the friction at the pivot. A gentle tapping of the compass will enable the needle to assume its proper position.

Note.—On account of the steel used in reenforced concrete construction, select, for this work, a table which is well away from the main walls and supporting posts of the building. Do not use a pencil which is encased in metal, until it has been tested and it is certain that it does not affect the compass needle. If the work must be done on two days, the same magnet should be used on each day and the work should be done at the same place in the laboratory.

### Questions

**1.** Do lines of force represent lines of equal force *i.e.*, does a particular line pass through points at which the same magnetic intensity exists?

**2.** Explain how the *direction* and the *magnitude* of the forces exerted by the poles of the magnet at any point in the field may be computed, if the strength and the positions of the poles of the magnet are known.

**3.** As a special case, draw a second diagram to explain the formation of the neutral points. Make the construction correspond to the position of the neutral points on the map.

## Part II

OBJECT: *To map the field of force due to a permanent bar magnet.*

A magnetized needle, in the earth's field, is acted upon by a torque (moment) which tends to bring the needle into the magnetic meridian.

### Questions

**1.** Make a diagram of the magnetic needle and the forces acting on its poles, which cause this torque. As the needle becomes more nearly parallel to the magnetic meridian, what change occurs in the value of this torque?

**2.** What is the value of this torque when the needle is in the magnetic meridian?

**3.** Suppose that a bar magnet is brought near a magnetic needle in such a way as not to disturb the latter's position in the magnetic meridian. What can be said as to the direction of the field due to the magnet?

With the aid of a large compass, stick two pins into the table so that a thread stretched between them is in the magnetic meridian. As before, place the magnet near the center of a large piece of paper, outline it, and indicate its polarity. Place the small compass near the north pole and then move the paper (with bar magnet in place) until a position is found such that the compass needle is directly under and parallel to the thread. Trace the lines of force as before, but each time the compass is moved ahead turn the paper so as to bring the needle parallel to the thread.

### Question

4. Explain why the method used in plotting this field eliminates the effect due to the earth's field.

*Optional*

5. Ask the instructor for a piece of soft iron, or another magnet to place on the paper a few centimeters from the first; plot the resultant field.

## EXPERIMENT 44

## Relative Determination of H

OBJECT: *To determine the ratio of the horizontal components of the earth's magnetic field at several places by using an oscillating magnet.*

If a magnet is supported at its center of gravity by a torsionless fiber, it will place itself parallel to the earth's field and, when displaced horizontally through a small angle, will vibrate in simple harmonic motion with a period whose value is determined by the expression

$$T = 2\pi \sqrt{\frac{I}{MH}},$$

$I$ representing the moment of inertia, $M$ the magnetic moment, and $H$ the horizontal intensity of the earth's field. If the same magnet is set in vibration at two stations, $I$ and $M$ will remain constant, and, consequently, the values of $H$ at the two stations will vary inversely as the squares of the periodic times. Accordingly, this serves as a simple method of comparing $H$ at different points on the earth's surface.

To determine $T$, set the magnet vibrating through a small angle by bringing a piece of iron or steel near it and record the time of the zero transit and of the tenth, twentieth . . . seventieth transits. Do this for three stations at one of which $H$ is known, and compute $T$ by the method explained in Par. 10 of the Introduction. Find the value of $H$ at the two unknown stations.

### Questions

1. Define magnetic moment and give its physical meaning.
2. Prove that the values of $H$ at two stations vary inversely as the squares of the periodic times.

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3. Show that the vibrating magnet is moving in simple harmonic motion.

## Descriptive Note

**Galvanometers.**—The galvanometers which are commonly used in measuring direct currents have one point of construction in common—they all possess a permanent magnet and a coil through which flows the current to be measured. The action of these two parts produces a deflection proportional to the current. They can be divided into two general classes: the moving magnet and the moving coil instruments. As the name indicates, in one, the magnet is a light needle which moves while the coil is fixed, which form is usually called the Thomson galvanometer. The other type has the reverse arrangement and is spoken of as the d'Arsonval. For ordinary work, the latter is much more convenient, because outside magnetic fields have very little effect upon it, but for extreme sensibility the other type must be employed.

*Measurement of Current.*—In most galvanometers there are always slight variations of the zero, but the effect of these variations can be largely eliminated if a deflection on each side of the zero is taken instead of on only one. The value of the single deflection can be obtained from these two readings without knowing the zero, if their average is taken. For the sake of accuracy, always use as large a reading as possible; a mistake of a millimeter will obviously cause a smaller percentage error in a large reading than in a small one. If it is necessary to know the magnitude of the current flowing in amperes, then the *constant of the galvanometer* must be known. This term, as used in this laboratory, means the fraction of an ampere necessary to produce a single deflection of 1 cm., so, if the single deflection produced by a certain current is multiplied by this quantity, the current in amperes is obtained.

*Measurement of Difference of Potential.*—A galvanometer can be used to measure potential difference as well as current if its resistance is known. For, by Ohm's law, this potential difference will be the current times this resistance. One very important consideration must, however, be noted in this connection. If the two points between which the potential difference is to be measured are the ends of a wire through which a current is flowing, the galvanometer circuit should be of much higher resistance than this wire, as otherwise it will lessen the resistance between the two points and hence, in general, the potential difference. Galvanometers used to measure potential should, therefore, have a high resistance.

*Shunt Boxes.*—To meet all the various requirements which are demanded of galvanometers in a laboratory, either a large number of such instruments covering a wide range of sensibilities has to be at hand or else some device must be employed which will permit one galvanometer to be used under quite different circumstances. In this laboratory an application of the principle illustrated in Exp. 48 will be employed to increase the range of each instrument. Below the galvanometer is a shunt box which has a set of connections as shown in Fig. 29*a*. *A* and *B* are binding posts attached to the shunt box. The wires *Ac* and *Bd* are connected to the galvanometer, and the wires *Ae* and *Bf* lead to the particular circuit in use. The post *A* is permanently attached to the bar *L*, and the post *B* has a wire running from it to the resistances 1, 2, 3, etc. If a plug is introduced so as to connect any one of the discs to the bar *L* it is obvious that the

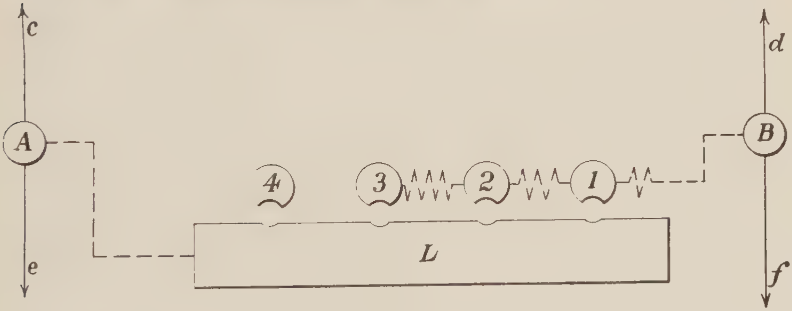


FIG. 29*a*.

resistance attached to that disc is thus placed in parallel with the galvanometer. Hence, by having the coils of varying resistances, the amount of current which flows through the galvanometer may be regulated at will. From the diagram, it is evident that, when the plug is in position 4, there is no shunt across the galvanometer, *i.e.*, it is at its highest sensibility.

Two defects are inherent in this arrangement: (1) The plug completes the shunting circuit and its resistance may well be comparable with that of the shunt, when the plug is in position 1. This makes the readings erratic, depending as they do upon the pressure with which the plug is inserted. (2) When the coil moves in a magnetic field, an e. m. f. is induced which sends a current through the shunt and damps the motion. With the low-resistance shunts used here, this current is so large as to make the motion very sluggish and reading tedious.

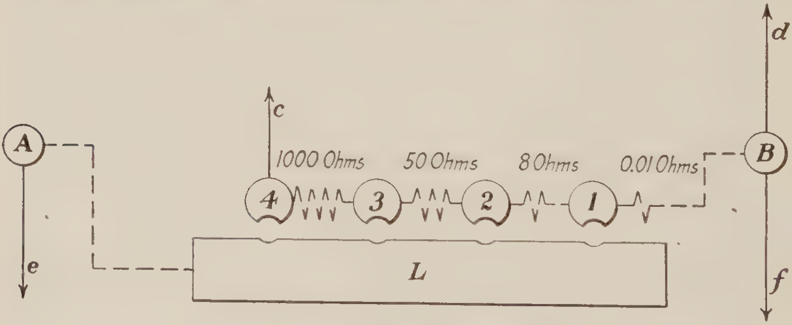


Fig. 29*b*.

To obviate these defects, an Ayrton type of shunt is used on many galvanometers. From Fig. 29*b* it will be seen that the galvanometer is permanently shunted by 1058 ohms. This shunt is so high that the sensibility is not much reduced and the damping is not excessive. A careful study of the figure will show how the first mentioned difficulty is (largely) eliminated.

The galvanometers in this laboratory have all been calibrated and their constants will be found tabulated on the bulletin boards. The resistances given refer to the galvanometer circuit as a whole, *i.e.*, galvanometer and shunt combined.

*Ammeter, Voltmeter.*—The ammeter is a d'Arsonval galvanometer modified in such a way as to make it portable. The coil itself is mounted in jewel bearings, and in place of the long phosphor-bronze suspensions there are two spiral springs which oppose the motion of the movable coil. To this coil is attached a pointer which indicates the amount of rotation, but, instead of reading the deflections in degrees, the scale is graduated to read directly in amperes. The voltmeter is similar in construction to the ammeter, but it is of very high resistance and has its scale calibrated to read in volts.

It is often convenient to have the same instrument cover two ranges of values instead of one. Figure 30*a* shows how some of the ammeters are arranged in this laboratory to accomplish this. If wires are connected to the binding posts, marked 2 and +, then as much current flows through the shunt as through the ammeter, for the resistances of the two are equal, so the current flowing is twice the reading of the instrument. When the posts marked 5 and + are used, then the current flowing is five times the reading of the ammeter. In general, however, in marking the range of an instrument, the multiplying factor is not given, but the number placed near one binding post indicates the maximum reading which can be obtained with that combination.

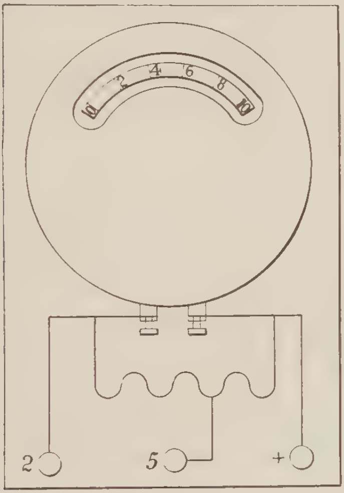


FIG. 30*a*.

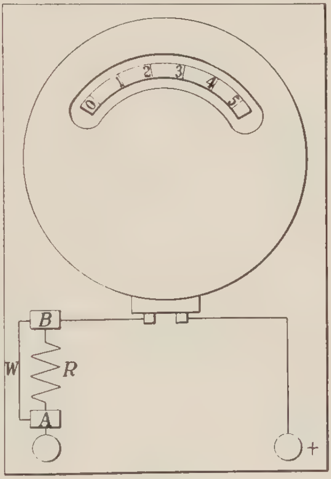


FIG. 30*b*.

Figure 30*b* indicates how a voltmeter must be connected to give two or more ranges. Suppose *R* has a resistance 29 times as great as that of the voltmeter. Then, if the voltmeter reads 5 volts, the resistance must have a potential difference of 145 volts between its terminals, thus making the total difference of potential 150 volts. Accordingly, the multiplying factor is 30. If the copper wire $W$ is introduced between the blocks $A$ and $B$, the resulting resistance is negligible and the voltmeter is direct-reading. In all of the other voltmeters used in the laboratory the maximum number of volts which can be measured is indicated by a number stamped on the case near one of the binding posts.

The preceding remarks apply particularly to the larger and more rugged type of ammeter and voltmeter found in the laboratory. In addition to these, there are a considerable number of smaller and more portable instruments. These are among the most easily injured instruments in the laboratory and special attention is called to the following caution: *Take great care that the current or voltage to be measured does not exceed the range of the instrument.* In particular, when using a double or triple-range instrument *be sure the connections are made with the right pair of terminals.* Thus, certain of the triple-range voltmeters have four binding posts marked, respectively, 3, 15, 150 and +. To measure 110 volts, say, connections should be made with the last two mentioned. To connect with 3 and + would be to throw 110 volts on a part of the instrument designed for a maximum voltage of only 3, and serious damage would be sure to result.

## EXPERIMENT 45

### Equipotential Surfaces

OBJECT: *To map the equipotential surfaces and lines of flow in a flat extended conductor.*

As the name indicates, an equipotential surface is one over which the potential is the same. Although the word *surface* is used in this connection, this experiment really deals—because of the thinness of the conductor—with merely the traces of such surfaces, *i.e.*, equipotential *lines*. With very slight modification, however, the same general method could be used to determine the equipotential surfaces in a three-dimensional conductor, *e.g.*, a liquid in a deep vessel. The lines of flow are the path along which the ions tend to move and, as the flow must at all points be perpendicular to the equipotential surfaces, their general form can also be found.

A thin (1 or 2 mm.) layer of tap water in a flat-bottomed glass dish will be used as an extended conductor. This is provided with electrodes connected to the secondary of a small induction coil—or, much better, a microphone "hummer"—which furnishes an alternating current, *i.e.*, one flowing first in one direction and then in the other, these reversals taking place many times a second. A telephone receiver is attached to two other electrodes, one of which is provided with a base, thus making it possible to place the electrode at any desired point in the liquid. If the other electrode, known as the "equipotential seeker," is moved about until the receiver makes no sound, the two points are on the same equipotential surface. Leaving the weighted electrode in one place and determining a number of such points with the other, the equipotential surface is definitely marked out.

To facilitate the location of these points, a sheet of plotting paper is put under the glass dish. See that the fixed electrodes which carry the current in and out of the liquid are on a line of the cross-section paper near the middle of the sheet. Complete the circuit, tapping the hammer of the coil with the finger, if necessary, to start it in vibration. If the hammer does not vibrate properly, open the switch at once or the battery will be quickly exhausted. Place the weighted electrode which is attached to the receiver on a point midway between the fixed electrodes; then move the potential seeker up and down lines parallel to the one joining the fixed electrodes until points are found where the receiver gives no sound. *Be sure that the seeker is always held vertically.* If there is much noise in the room, it is often difficult to determine such points sharply. In that case locate the point by approaching first from one side and then from the other and use the average setting.

Determine five such points on each side of the middle line, *paying especial attention to the direction of the lines at the edge of* *the dish.* To do this, locate, one of the five points at the edge of the dish and another at about 1 cm. from this. A still better way is to move the weighted electrode—for these end points—to within a few millimeters of the edge or corner of the dish and locate the point of same potential with the seeker right at the edge or in the corner. Shift the weighted electrode so as to obtain three surfaces between the first one mapped out and each fixed electrode, determining at least 10 points on each surface. Thus seven equipotential surfaces should be mapped. As the positions of these points are located, transfer them to a sheet of plotting paper on which are indicated the outlines of the vessel and the fixed electrodes. Sketch in the equipotential surfaces. Indicate by dotted lines the lines of flow.

This experiment may be modified by introducing into the electrolyte a block of metal or glass. If such a piece is used, trace carefully the distribution of potential in its immediate neighborhood. If a strip of metal is used, place its long axis parallel to the line joining the two fixed electrodes; the glass should have its long axis at right angles to this line.

### Questions

**1.** Explain why the above method gives equipotential surfaces.

**2.** Why is it necessary to use an alternating current?

**3.** What angle do the equipotential surfaces make with the side of the dish? Why?

## EXPERIMENT 46

## Electrolysis

Object: *To check the accuracy of an ammeter by means of the copper coulometer.*

Faraday's laws of electrolysis make it possible to determine accurately the average current flowing in a particular circuit by measuring the amount of metal deposited by the current in a given time. Hence this method may be used to calibrate a current-measuring instrument as, for example, an ammeter, *i.e.*, to determine the currents which produce certain readings on the instrument. This method is extremely accurate but tedious, for it takes nearly an hour to make one determination, and to calibrate the instrument completely would necessitate at least ten such determinations, for different currents within the range of the instrument. In the present experiment only one or two determinations will be made.

The circuit is to be connected according to Fig. 31. *B* is a storage battery, *R* an adjustable resistance, *A* the ammeter, and $V_1$ and $V_2$ coulometers. The form of the coulometer used here consists of a battery jar, on the cover of which the electrodes are mounted, so that they can be easily removed from the solution for washing. The two outer plates serve as anodes and the inner as the cathode.

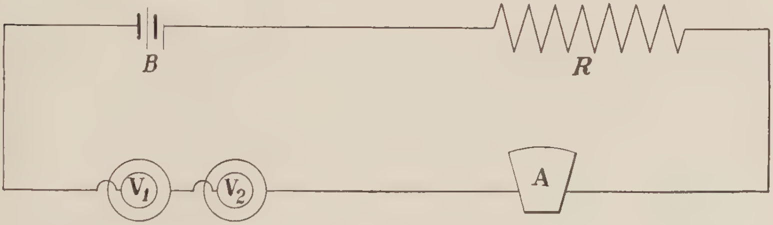


FIG. 31.

Select the cleanest of the three plates and insert it in the center as the cathode, and then make a preliminary run of a few minutes. This enables one to get acquainted with the apparatus to determine the adjustment of the rheostat to give a suitable current, and also puts a fresh, clean deposit of copper on the cathode. Be sure that the connections are correctly made so that the current is running in the right direction through each coulometer, and, before throwing in the switch, ask the instructor to check over the wiring. The current to be used may be computed on the basis of 20 milliamp. per square centimeter of cathode surface (remember that the plate has two sides). After allowing the current to flow for a few minutes, remove and rinse the plates; wash the cathode very carefully at the sink, and dry it by holding it about 2 ft. above a Bunsen flame.

Having gained some acquaintance with the apparatus, the experiment may now be performed. Weigh the cathode very carefully; then assemble the apparatus and allow the current to flow for, say 20 min., measuring the time to a second. During this time record the ammeter reading once a minute. Remove and wash the plates as before; dry and weigh carefully. Assuming the electrochemical equivalent of copper to be 0.000329 gm. per coulomb, compute the average current and compare with the average ammeter reading. Tabulate the results.

**Precautions:**

1. Read and follow the directions given at the end of Exp. 3 for the use of the balance.

2. Take all readings (masses, times, currents) as accurately as possible, in the case of the latter holding the eye directly over the ammeter needle.

3. Do not handle surfaces of electrodes. If cathodes show oxidation, they may be cleaned with sandpaper.

4. Make and break the current at the switch; do not leave the electrodes in the solution when current is not flowing. A beaker of water should be at hand in which they should be rinsed *immediately* after removal. Wash the cathode in running water. The electrolyte (acidulated solution of copper sulphate, density about 1.12) should be at the proper height in the jar—not high enough to touch the plate clamp, but yet covering the main part of the plates.

5. Return the electrolyte to the bottle when the experiment is completed; wash the jar and plates. Be sure the analytical weights are in their proper places and return them and the coulometer to the desk.

### Questions

1. Is the coulometer a *current*- or a *quantity*-measuring instrument?
2. Which reading (mass, time, current) would it be desirable to take more carefully if possible?
3. Why is it desirable to have two anodes in this case?

## Descriptive Note

**Ohm's Law.**—The usual statement of Ohm's law,

$$I = \frac{E}{R},$$

is ambiguous, and, when this formula is used, care must be taken to interpret correctly the symbols. A more precise way of stating the law is to express it in at least two forms; one, when the entire circuit is considered, and the other, when only a portion not containing an e. m. f. is taken into account. In the first case $E$ stands for the total electromotive force, $R$ is the total resistance of the whole circuit, and $I$ is the current furnished by the battery. When a part of the circuit is to be considered, $E$ is the difference of potential existing between the two points which limit the portion under consideration, $R$ is simply the resistance between two points, and $I$ is the current flowing from one point to the other through the resistance $R$. The resistances and the currents flowing in the rest of the circuit do not have to be considered. For example, the difference of potential between the points $A$ and $c$ in Fig. 34 may be determined in three different ways. It is equal to the current flowing through the wire $AB$ times the resistance of the wire between the points $A$ and $c$; it is equal to the current flowing through the galvanometer circuit times the resistance in that branch; or it is equal to the current which flows up to the point $A$ multiplied by the actual combined resistance of the wire and the galvanometer circuit. As far as the result is concerned, it is immaterial which one of the three methods is selected; but usually only sufficient data are at hand to solve the problem by one of them.

## EXPERIMENT 47

## Determination of Resistance and E. M. F. of a Cell

OBJECT: *To determine the internal resistance and e. m. f. of two batteries by Ohm's method.*

Ohm's law states that the current flowing in a circuit is equal to the difference of potential between two points, divided by the resistance between those same points, or it is equal to the e. m. f. of the circuit divided by the total resistance. If a galvanometer, a battery, and a resistance box are connected in series, then the current flowing is obtained from an expression of the form

$$I = \frac{E}{R + R_b + R_g},$$

$R$, $R_b$, and $R_g$ being the resistances of the box, the battery, and the galvanometer respectively. Hence, if these resistances and the corresponding currents are known, the e. m. f. of the cell may be computed. This forms the basis for the present experiment. Before starting this work, read the discussion of galvanometers following Exp. 44. The instructor will specify the particular shunt to be used in this experiment.

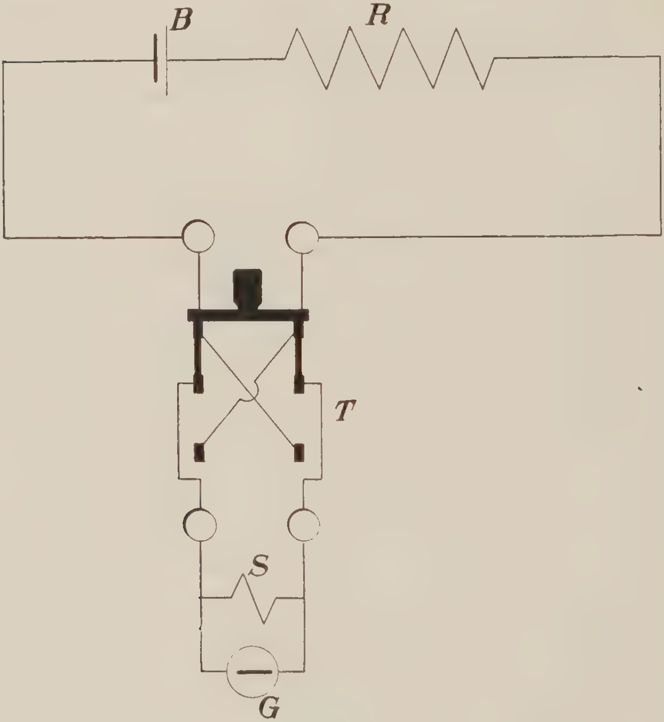


FIG. 32.

Connect according to Fig. 32. $B$ represents the battery, $R$ the resistance box, $T$ the reversing switch, $S$ the shunt box,

$G$ the galvanometer. Make two runs: first with a gravity cell and then with an Edison Lalande (or other battery which may be specified, *e.g.*, a dry cell), taking in each case direct and reverse readings of the galvanometer for resistances 4, 6, 9, 12, and 15 ohms in the box. Always take direct and reverse readings for each resistance and be sure that the plugs of the resistance box fit snugly. Do not push the plugs into place by main force, but twist them into position using moderate pressure.

When currents of a certain magnitude are furnished by an Edison Lalande, its e. m. f. changes. Consequently, it is well to start with 15 ohms when working with this cell and go down the series of resistances. It will be found that for the smaller values the current changes constantly. Hence the readings should be taken quickly, and, if the galvanometer should show a tendency to drift, it should not be allowed to come to a complete rest, but a reading should be taken as soon as the coil stops vibrating and this drift appears. The switch should then be immediately opened.

From the deflection of the galvanometer, the current may be computed in amperes, because the constant of the galvanometer has been determined and is posted on the bulletin board. The meaning of the term "constant of a galvanometer," is given in the discussion following Exp. 44, in the paragraph which describes the measurement of current. The value of $R_b + R_g$ can be read from the curve and the e. m. f. of each cell determined. One computation is to be made for each cell, data being used which correspond to a point which lies on the curve. Since the galvanometer resistance (for the particular shunt used) is posted with the galvanometer constant on the bulletin board, the battery resistance is found at once.

Determine the e. m. f. and the resistance of each of the two cells.

**Curves.**—Plot on the same sheet, one curve for each run, using the values of $R$ as abscissas and the reciprocals of currents as ordinates. It will be found most convenient to use the 20-cm. side of the paper for the abscissas and displace the origin 5 cm. from the lower left-hand corner along the axis of abscissas. A little study should make it clear how the value of $R_b + R_g$ can be obtained from these curves. The table of reciprocals at the end of the book will materially assist in these computations.

### Questions

**1.** Show that the curves are in agreement with Ohm's law.

**2.** Explain how the value of $R_b + R_g$ may be obtained from the curve.

**3.** Which battery has the larger resistance? Explain how the data of the experiment show this.

**4.** Explain why for certain values of $R$ a battery having a lower e. m. f. is still capable of furnishing the larger current.

## EXPERIMENT 48

## Theory of Shunts

OBJECT: *To study the way in which changes in the resistance of one of two conductors in parallel affect the current in the other.*

Two pieces of electrical apparatus are said to be connected in parallel when the current which flows up to them divides, part going through one, and part through the other. The same form of connection is meant when the pieces are said to be connected in multiple or one is said to be shunted around the other.

Make the connections as shown in Fig. 33. $B$ is the battery, $R$ a large resistance, $r$ a box giving small values, $T$ reversing switch, $G$ the galvanometer, $S$ the shunt box. Make the resistance in $r$ infinite and adjust $R$ so that a single deflection of the galvanometer equals about 15 main scale divisions, *i.e.*, centimeters. When this adjustment has been made, change $R$ by 5 ohms and see if the reading of the galvanometer is altered. This observation will aid in answering question 5. When this value of $R$ has once been found, do not change it while taking the readings of the galvanometer corresponding to the various values of $r$. Then take direct and reverse readings of the galvanometer for the following resistances of $r$: infinite resistance, 0, 1, 2, 3, 4, 5, 7, 10, 15, 20, 40 ohms. The plugs in the resistance box should be in tight; this can be tested at the beginning of the run, for, if there is a deflection of more than 2 or 3 mm. when $r$ is zero, it means that some plug is loose, and it should be twisted securely into place. Since the galvanometer is so constructed that the current flowing through it is directly proportional to the deflection, in plotting, single deflections of the galvanometer may be used for current.

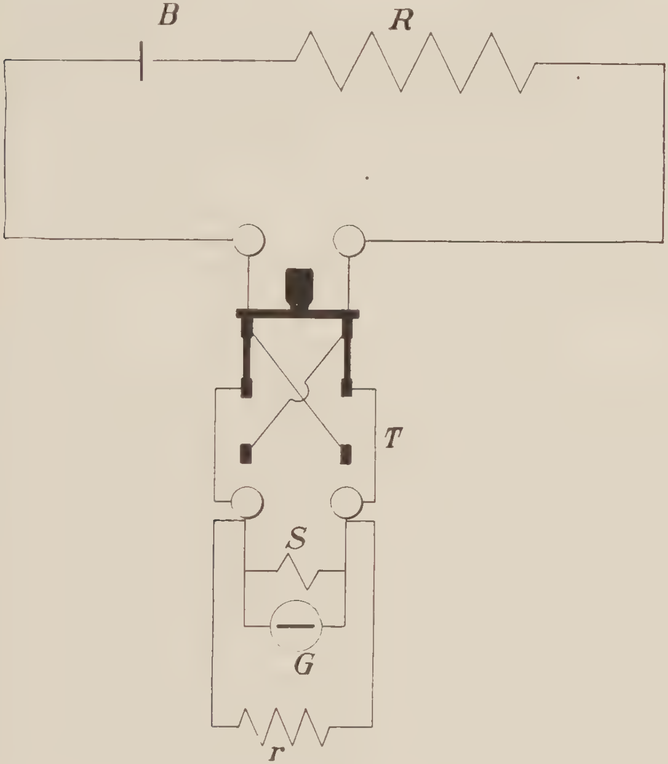


Fig. 33.

**Curves.**—Plot a curve, using deflections as ordinates and corresponding values of $r$ as abscissas. The circuit is so arranged that the current which is divided between $r$ and the galvanometer circuit is constant for all values of $r$. Decide which of the above readings represents this total current and draw a line parallel to the axis of abscissas and at a distance from it equal to this value.

### Questions

**1.** State how the current is distributed between two conductors in parallel.

**2.** Intreprct the curve *i.e.*, give the physical meaning of the ordinates from the axis of the abscissas to the curve and from the curve to the horizontal line.

Show how the curve is in agreement with the theory of shunts. Find the resistance of the galvanometer circuit from the curve.

**3.** Explain the shunt box attached to the galvanometer, using a diagram. A discussion of the shunt box is given in the descriptive note following Exp. 44.

**4.** Compute the currents furnished by the battery when $r$ is zero and when $r$ is infinite. In order to do this, assume the e. m. f. of the battery to be 1 volt and its internal resistance to be 3 ohms. Substitute for $R$ the value used in the experiment.

**5.** The current furnished by the battery is assumed to remain constant throughout the experiment. State the basis for this assumption.

## EXPERIMENT 49

## Distribution of Potential along a Wire Carrying Current. Theory of the Voltmeter

OBJECT: *To study the variation of potential along a wire carrying a current; also to determine the effect on such measurements of a change in the resistance of the measuring instrument.*

When a current flows through a uniform wire, the potential has a relatively simple distribution; in the present experiment, this distribution will be determined first by a high-resistance galvanometer circuit and then by one of low resistance, to see if the change in resistance of the galvanometer circuit has any effect upon the values obtained.

A circuit of the following form will be used. Between the points $A$ and $B$ (Fig. 34) is stretched a fine wire of relatively high resistance. The battery consists of a gravity cell, and $R$ is a moderately high resistance. $S'$ represents the shunt box and $R'$ a variable resistance which is to be set at 30 ohms for the first run, using galvanometer shunt 3. Place the movable contact as far from $A$ as is possible and adjust $R$ until a single deflection of about 15 scale divisions is obtained. Then take direct and reverse readings of the galvanometer when $c$ is at 0, 20, 40, 60, 80 and 100, as indicated on the scale parallel to the wire. Repeat these readings in a second run, using shunt 4 and adjusting $R'$ to give about a 15-cm. deflection. Keep the value of $R$ the same during the whole experiment, but use two values of $R'$, one with each shunt, which will give the deflection mentioned above. If the total resistance of the galvanometer circuit is known, an application of Ohm's law, as explained just before Exp. 47, will give the difference of potential between $A$ and the point $c$, for the current flowing through the galvanometer circuit can be determined from the deflection and the constant of the instrument. Compute these differences for all the readings.

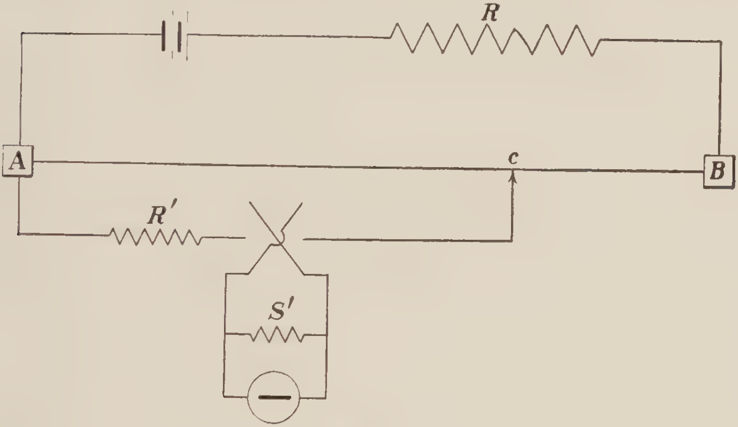


FIG. 34.

**Curves.**—Plot two curves, both on one sheet to the same scale, using lengths of wire as abscissas and potential differences, obtained in the different sets of readings, as ordinates.

### Questions

**1.** Interpret the curves, explaining why one is straight, and the other curved, paying special attention to the physical meaning of the divergence of the curves.

**2.** What kind of galvanometer, *i.e.*, high or low resistance, is needed for measuring differences of potential? Explain briefly the construction of the voltmeter.

## EXPERIMENT 50

### Distribution of Potential along Several Wires Connected in Series

OBJECT: *To determine the potential distribution along several wires in series.*

The drop in potential along a wire is a function of its resistance and the current flowing; hence it will vary in wires of different sizes and materials which carry the same current. In this experiment the fall in potential over several different wires will be studied, the wires being arranged on a board so that a galvanometer may be readily shunted between any points. Three of these wires are of the same size but of different materials, one being copper and the other two alloys. The fourth wire is of copper, but of much smaller size. Set up the circuit according to Fig. 35. Use one gravity or dry cell and make $R$ 500 ohms.

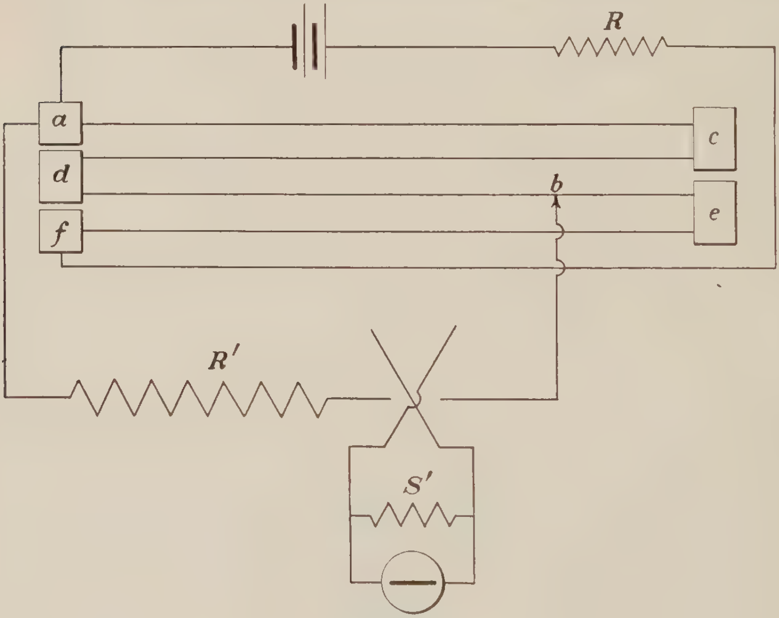


FIG. 35.

Connect one terminal of the galvanometer circuit to the point *a* and move *b* as near to *f* as possible. Then adjust *R′* until a single deflection of about 15 scale divisions is obtained. The shunt S′ will be designated by the instructor. Take direct and reverse readings with *b* at 0, 30, 70, and 100 cm. from *a*. Take similar readings in going from *c* to *d*, *d* to *e*, and *e* to *f*. The constant of the galvanometer and its resistance should be looked up (see bulletin board), and from the former the currents flowing in the galvanometer circuit may be computed. By an application of Ohm's law as explained just before Exp. 47, the potential in volts may be determined for the various readings.

**Curves.**—Plot a curve, with lengths of wire as abscissas and the potential differences as ordinates. The length corresponding to a particular reading is the distance from *a* to the point in question. Thus the distance for *e* equals the total lengths of the three wires from *a* to *e*.

### Questions

**1.** How does the potential vary along a single wire?

**2.** Interpret the curve. Which wire has the greatest resistance? Which the least? Give reasons for these statements.

**3.** What kind of galvanometer, *i.e.*, high or low resistance, should be used in measuring differences of potential?

## EXPERIMENT 51

## Arrangement of Cells

OBJECT: *To study the way in which the e. m. f., resistance and current furnished by a battery depends on the arrangement of the cells.*

Cells making up a battery may be arranged in three ways: all in series, all in parallel, or a combination of series-parallel connections. While it would be desirable to have a battery of a great many cells with which to perform this experiment, the main facts may be illustrated with even as small a number as four. Test four gravity cells to see if their e. m. fs. are practically the same, and, if they are, connect them in series with a resistance box and an ammeter, and in parallel with a voltmeter, using a switch, so that the current will flow only while the readings are being taken. Figure 36 shows the connections and indicates the series-parallel arrangement of the battery. For each of the required resistances, read the ammeter and the voltmeter. Before starting to make any observations, read the description of ammeters and voltmeters given just before Exp. 45.

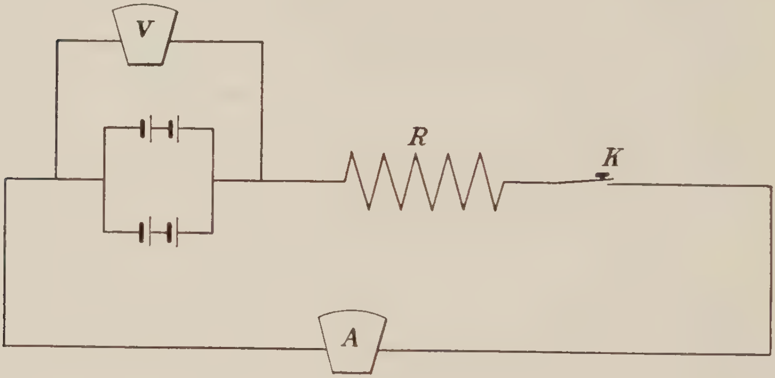


Fig. 36.

The following observations are to be made:

| Arrangement of cells: | Resistances to be used: |
|---|---|
| All in series | 0, 2, 7, 10, 15, 20, and infinity |
| All in parallel | 0, 1, 2, 3, 4, 7, 10, 20 and infinity |
| Series parallel | 0, 1, 2, 4, 7, 10, 15, 20, and infinity |

Have the data looked over by an instructor before taking down the apparatus.

**Curves.**—Plot two sets of curves. Use one sheet for each set and plot the curves to one set of coordinates.

1. Resistances as abscissas and currents as ordinates.
2. Currents as abscissas and potential differences as ordinates.

### Questions

**1.** What effect upon the e. m. f. of the battery have the various arrangements?

**2.** Explain distinction between e. m. f. of cell and potential difference across its plates. Under what conditions do these differ numerically? Why?

**3.** Compute the battery resistance for each arrangement, using the expression $P = E - R_b I$. $P$ represents the potential difference at the terminals of the battery when it is furnishing the current $I$, $E$ the potential difference when no current is flowing, $R_b$ the resistance of the battery. Make two computations for each arrangement, using values which lie on the curves. How do the different arrangements affect the resistance of the battery?

**4.** From the first set of curves, state the range of resistances which causes each arrangement of cells to give a current larger than that furnished by either of the other combinations. It can be proved mathematically that, for a given external resistance, a given number of cells will furnish the maximum current when they are so connected that the battery resistance is most nearly equal to the given external resistance. Show that the curves obtained are in general agreement with this statement.

## EXPERIMENT 52

## Practical Electricity

OBJECT: *To measure in two ways the power expenditure in some piece of electrical apparatus, e.g., an electric flat iron, a lamp bank or electric hot-plate; also to gain some acquaintance with ordinary electric connections.*

### Part I

Electrical power, like other sorts, is measured in watts, and energy in joules ("watt-seconds"). The common electrical unit of energy, however, is the *watt-hour*. Every electric-lighted house has a watt-hour meter, and the monthly bill is rendered for so many *kilowatt-hours* of energy consumed.

In the present case, the power consumed by an electric iron, hot plate or a bank of lamps will be measured with the aid of a recording watt-hour meter and also by readings of an ammeter and volt-meter in the circuit. Read the description of ammeters and voltmeters given just before Exp. 45, paying particular attention to the cautions of the last paragraph.

After answering questions 1 and 2 below, decide how the ammeter (0 to 5 amp.) and the voltmeter (0 to 120 volts) should be arranged in the circuit, and connect them accordingly, but *do not close the switch until the connections have been checked by the instructor.* As there are 116 volts on this circuit, a mistake in connecting is likely to ruin one of the instruments.

While the iron is rated at 550 watts, it will become very hot if allowed to run continuously at this rate while on the stand, so the current is cut down by a rheostat, which should be adjusted (only if necessary) to give a current of a trifle over 4 amp. The lamp bank will take about 360 watts and no resistance will be necessary.

While the lamps heat up almost instantly, the iron must heat for about 5 min. to reach a steady temperature, before any final readings can be taken. In the meantime, study the recording watt-hour meter. It will be found that the recording dials at the top run so slowly that they will not allow an accurate measurement of the energy consumption in this experiment unless it is continued for many hours. The revolutions of the aluminum disc are easily counted, however, and the disc in this type meter makes *four revolutions* for 1 *watt-hour*. Time 100 revolutions of the disc and compute the power consumption.

Make two complete runs, in each case measuring the power consumption by the voltmeter and the ammeter, and also by the watt-hour meter. Assuming the voltmeter and the ammeter to be correct, determine the percentage error of the watt-hour meter.

### Questions

(Answer 1 and 2 before performing the experiment.)

**1.** From the definition of power as the energy consumed per second, decide whether the watt-hour is a *power* or *energy* unit. How many joules in a kilowatt-hour?

**2.** Draw a diagram showing how the ammeter, voltmeter, and iron or lamps are connected in the circuit. If a mistake should be made and the voltmeter connected in place of the ammeter and *vice versa*, which instrument would be injured?

**3.** If electrical energy costs 6 cts. a kilowatt-hour, how much would it cost to run the iron or lamps, as they were used, for 3 hr.? To run a 60-watt lamp for 8 hr.? A 40-watt fan 24 hr.? A $\frac{3}{4}$-kw. electric range "burner" 2 hr.?

**4.** Seven hundred forty-six watts equal 1 hp. What will it cost to run a washing machine requiring $\frac{1}{4}$ hp., for 3 hr.? What fraction of a horsepower does an iron taking 550 watts consume?

**5.** Study the dial system of the watt-hour meter. Make a reading and have the instructor check its accuracy.

### Part II

The instructor will furnish special apparatus for this part of the experiment and the student is to connect up an extension cord with plug and socket, an ordinary snap switch with light and a 3-way switch combination with light. Instructions will be found with the apparatus. After completing this part of the experiment have connections checked by instructor and then disconnect all wires so that apparatus is in the same condition in which it was found.

## EXPERIMENT 53

## Measurement of Resistance by Voltmeter and Ammeter Method

OBJECT: *To determine the resistance of a carbon and of a tungsten lamp by means of a voltmeter and an ammeter.*

From Ohm's law, $R = \frac{E}{I}$, it is seen that, if a potential difference $E$ causes a current $I$ to flow in a resistance $R$, the measurement of $E$ and $I$ gives $R$ at once. The most convenient way of measuring these quantities is by voltmeter and ammeter. Please remember first of all that these are delicate instruments, which must be handled with care. Read the discussion of ammeters and voltmeters just before Exp. 45, particularly the last paragraph. Connect the circuit as shown in Fig. 37, but leave the switch open until an instructor has checked over the connections. Study the construction of the lamp-bank variable resistance and note how the lamp connections may be varied from series to parallel, as well as a combination of the two methods. By this means a wide variation of resistance and a corresponding variation in the current through the lamp under test may be obtained. In some of these connections only a very small current will flow through the lamps, so do not expect them to light under all conditions.

Make two runs, one for a carbon and one for a tungsten lamp of about the same candlepower, using eight currents in each case. Choose these currents *so as to give well-distributed points;* the smallest current should be about one-tenth of the largest. For each setting of the variable resistance read the current in amperes through the lamp and the potential in volts across its terminals. Remember that the ammeters and voltmeters are not necessarily direct-reading; when used with an external shunt, as they frequently are in the ammeter, it will be necessary to multiply their indications by the appropriate factor. Use the connection on the instrument which gives the largest reading in each case, as the larger the reading the smaller the percentage error. If the instruments do not read zero when no current is flowing, corrections must be made for this "zero error."

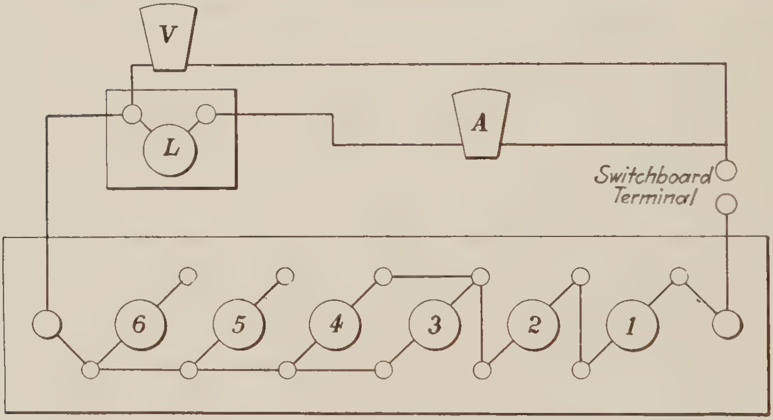


FIG. 37.

**Curves.**—Plot on the same sheet two curves, one for each lamp, with currents as abscissas and resistances as ordinates.

### Questions

**1.** What do these curves show as to the relation between the resistance and the temperature in the case of each lamp? In answering this remember that, in general, the larger the current flowing the higher will be the temperature of the filament.

**2.** Would the change in resistance be due merely to the fact that current is flowing in the conductor (filament) or to some change in its physical condition, *e.g.*, temperature?

**3.** What is the essential difference between an ammeter and a voltmeter? Which has the larger resistance? Which would be most liable to injury if a mistake were made in connections?

**4.** Notice that in the foregoing experiment the voltmeter reads the potential drop across both ammeter and lamp. Under what conditions would this introduce a considerable error? Which of the two arrangements shown in Fig. 38 should be used in measuring an unknown resistance of low value? Of high value?

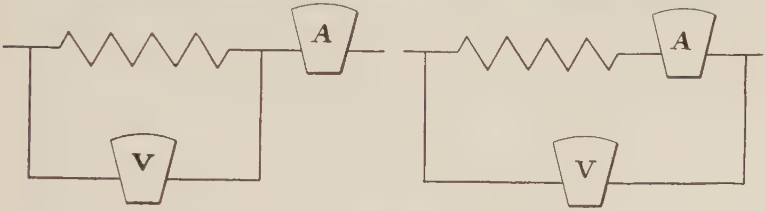


FIG. 38.

### EXPERIMENT 54

## Measurement of Resistance. Slide-wire Bridge

OBJECT: *To measure the resistances of two coils, when taken singly and also when connected in series and in parallel, by means of a Wheatstone bridge of the slide-wire form.*

The most convenient, and at the same time accurate, way of measuring resistances of widely different values is by means of the Wheatstone bridge, and its simplest form, the slide-wire bridge, will be used in this experiment. It is given this name because two of the resistances are formed by a wire of uniform cross-section, the dividing point between the resistances being a sliding contact. In connecting the bridge, time will be saved if the following method is used: At first pay no attention to the galvanometer, but consider only the four resistances and the battery. From Fig. 39 it can be seen that the current from the battery flows up to *a* and there divides, part flowing through $R_1$ and $R_2$ and the rest through $R_3$ and $X$. These two currents, however, unite at *c* and return to the battery. It makes no difference whether the slide wire represents $R_1$ and $R_2$, or $R_1$ and $R_3$, although the former is the customary arrangement, with the point $b$ corresponding to the slider contact. $X$ is the unknown and the remaining resistance must stand for the known, which is in the form of a variable resistance box. After the resistances are connected, insert the galvanometer as indicated. The keys in the battery and the galvanometer circuits are mounted on one base. The wires should be so connected that the key in the battery circuit is closed before that in the galvanometer circuit.

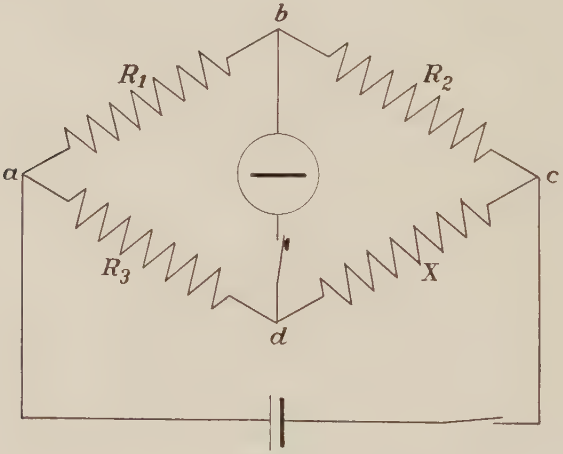


Fig. 39.

Place the slider near the middle of the wire and, starting with 1 ohm as the known resistance, note the direction of the deflection; then try 100 ohms and see if the opposite deflection is obtained. If it does reverse, this means that the unknown has a resistance of less than 100 ohms. Reduce the known resistance until a change of 1 ohm causes the galvanometer to change its deflection from one side to the other. Now move the slider until no deflection of the galvanometer is produced. When measuring an unknown resistance, it is well to start with a shunt of low resistance, Nos. 1 or 2, for example, and, as the deflection becomes smaller, to change to a more sensitive shunt. Do not record a balance point for each shunt, but use the various shunts to make one determination, entering on the data sheet the setting made with the most sensitive shunt used. In each case after a balance has been obtained and the data recorded, *vary the resistance in the box by a few ohms and rebalance the bridge, thus making two determinations for each unknown.*

The values of two unknowns are to be found singly, in series, and in multiple, making two determinations for each arrangement. Compute the values of the series and parallel arrangements and compare the experimental values with the computed.

In using the bridge, do not be misled by a false balance, which is due to poor contacts between the slider and the wire. When a balance has been obtained, take the readings; move the slider a centimeter or two from this point and note the direction of the deflection. Then move the slider a like distance to the other side of the supposed balance point. If the direction of the deflection has not reversed, the bridge is not in adjustment, and the reading is in error.

### Questions

**1.** Why is it better to have the slide in the center of the bridge instead of near one end when the balance is obtained?

**2.** Name three sources of error.

**3.** Make a diagram of the bridge, showing the *actual connections.* Indicate the points corresponding to *a*, *b*, *c*, and *d*.

## EXPERIMENT 55

## Measurement of Resistance. The Post-office Bridge

OBJECT: *To become familiar with the use of the post-office bridge for measuring resistance.*

The post-office bridge is a common and most convenient form of the Wheatstone bridge. It consists of a resistance box so arranged as to contain the ratio arms of the bridge and the known resistance, or, in other words, three arms of the bridge are contained in one box. Binding posts are attached so that the unknown, the battery, and the galvanometer may be connected at the proper points. The coils forming the two ratio arms usually have the values 1, 10, 100, 1000 ohms, and, in measuring a resistance, a definite ratio is used and the known varied until the bridge is balanced. In measuring the unknown, the best ratio to use first is that of unity, *i.e.*, 10:10 or 100:100, etc. In this case a perfect balance cannot usually be obtained, especially if the galvanometer is at all sensitive. The best that can be done, under these circumstances, is to find a value of the unknown which will produce a deflection in one direction and, when the next higher value is used, the deflection will be reversed. Thus the unknown is determined to the smallest unit in the known resistance. In the bridges in this laboratory, the value of the known can be found only to 1 ohm with equal ratio arms. By making the ratio 1:10, 1:100, etc., the unknown may be measured accurately to a small fraction of an ohm.

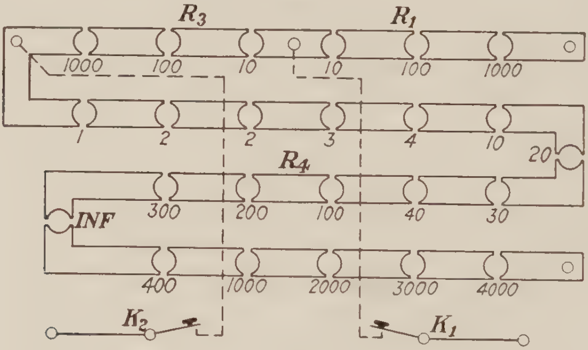
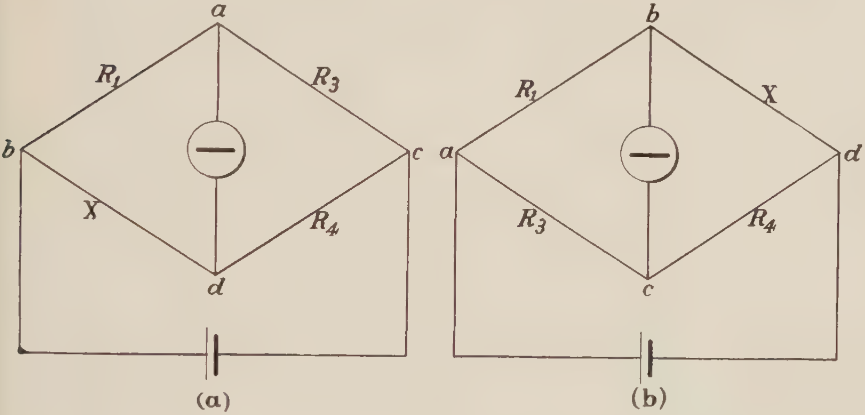


FIG. 40.

The principal error in using a box bridge is the contact resistance caused by the plugs not fitting firmly in the sockets. In putting the plugs in place, too great force is entirely unnecessary and often gives poor contacts. The best results are obtained by using *moderate pressure* accompanied by a twisting motion. *If, when using a higher ratio, the reading does not agree with the preceding, it usually means that the plugs have not been properly placed, and all the readings should be checked.* Another common error is introduced if the galvanometer key is closed before the battery key, for, if this is done, a deflection of the galvanometer may be obtained even when the bridge is balanced. If the keys are closed in the reverse order, no trouble will be encountered.

Figure 40 shows the top of the bridge used in this laboratory; the dotted lines represent conductors placed within the box itself so as to connect the points indicated. Connect the bridge according to either one of the diagrams given in Fig. 41, but specify which one is used. The simplest way to make the connections is to leave the galvanometer until the last. Starting with one wire from the battery, connect it with the box, so that one of the two keys is used and note that, after passing through the key, the circuit must divide; follow one of these branches and complete its connection back to the battery; do likewise for the other branch. Then it will be obvious where the galvanometer is to be attached.



(a) (b)

Fig. 41.

Determine the resistance of two unknowns, singly, in series, and in multiple, using ratios 1:1, 1:10, 1:100. By making the ratio 1:10, the known resistance must be 10 times as large as the unknown in order to secure a balance; hence the unknown may be determined to the nearest tenth of an ohm. By changing the ratio to 1:100, the next decimal place in the value of the unknown may be found. Care should be taken to make the ratio such that the known resistance will be larger than the unknown and not the reverse. At first, use the galvanometer with a shunt of low resistance which will not make it very sensitive.

When the deflection becomes too small, change to a shunt of higher resistance. It is unnecessary to record the determinations with all the various shunts, but enter on the data sheet only the value obtained with the most sensitive shunt. Compute the values for the series and the parallel arrangements and compare with the experimental values.

### Questions

**1.** Prove from Ohm's law that, when no current flows through the galvanometer,

$$R_1 : R_3 :: X : R_4$$

**2.** Derive the expression for resistances in parallel.

**3.** Make a drawing of the bridge, showing the *actual connections*, and indicate the points *a*, *b*, *c*, and *d*. Include also the schematic drawing.

## EXPERIMENT 56

## The Potentiometer

Object: *To study the potentiometer; in particular, to determine with it the e. m. fs. of several cells and to study the way in which the potential at the terminals of a cell depends on the current it is furnishing.*

Within the last few decades a most important instrument has been developed, called the potentiometer, which is capable of measuring accurately and conveniently differences of potential and, by the use of proper accessories, other electrical quantities.

Figure 42 shows the connections and will aid in explaining the principle. *D* is a storage cell which keeps a current flowing through the wire *AB*. Two cells *E* and *F*, whose e. m. fs. are to be compared, are connected as indicated in the figure, the switch *S* making it possible to place first one and then the other in series with the galvanometer. The potential difference existing between *A* and *B* must be greater than the e. m. f. of either *E* or *F* and must tend to send current through the galvanometer in a direction opposed to that of the two cells. Under these conditions it is possible to find a position of the sliding contact $c$, which will give no current through the galvanometer. Then the potential difference between $A$ and $c$ is equal to the e. m. f. of the cell which is in the circuit. Let $c$ be the position of the contact for the cell $E$ and $c'$ for $F$; then it can be proved that

$$\frac{\text{E. m. f. of cell } E}{\text{E. m. f. of cell } F} = \frac{Ac}{Ac'}.$$

The resistance $R$ is used to prevent too large a current being drawn from the cells $F$ and $E$, which might cause a change in the e. m. f. This coil has a resistance of 10000 ohms.

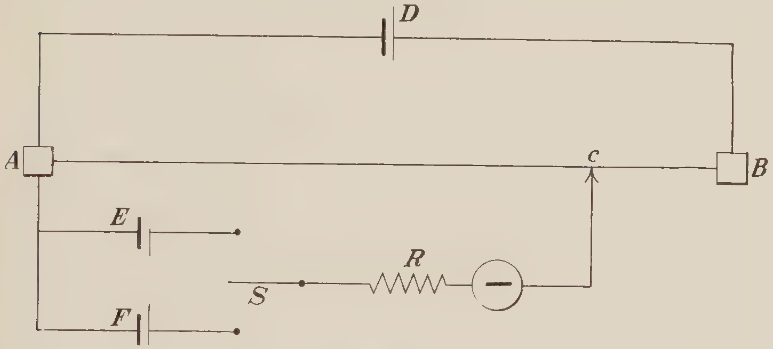


FIG. 42.

It is advisable to start with the shunt 1 or 2 of the galvanometer, depending upon sensibility, and, as the slider approaches the point of balance, change to a more sensitive shunt, if necessary. When starting to find a new balance, always return to the former shunt This does not mean that separate determinations are to be made with shunts 1, 2, 3, and 4, but that the various shunts are used to make the one setting as accurate as possible. One of the batteries is a standard cell of known e. m. f. and its value is marked on the case. Such a cell has to be handled with care and the student should specially guard against short-circuiting, as this renders it useless for several hours afterwards. It is advisable to make all the connections and see that the circuit is in working order before the standard cell is connected.

This experiment consists of two distinct parts, one or both of which will be required:

1. In this part the e. m. fs. of three cells are to be determined. To do this, let $E$ be a standard and $F$ a gravity cell. Take alternate readings for the two cells, starting with $E$, until, in all, five readings have been recorded, *i.e.*, three readings for $E$ and two for $F$. Determine the e. m. f. of the gravity cell. Also find the e. m. f. of an Edison Lalande cell and of a dry cell.

2. The second part consists in observing the potential difference at the terminals of the battery as it furnishes currents of different magnitudes. Connect a resistance box in parallel with a gravity cell and take data as follows: first a reading for $E$; then readings for $F$ when the resistance in parallel equals 40, 15, 5, 3, 1 ohms; and, finally, a second reading of $E$.

**Curve.**—(Part 2 only.) Plot a curve, with values in the resistance box as abscissas and potential differences at the terminals of $F$ as ordinates. Draw a line parallel to the $X$-axis and at a distance from it equal to the e. m. f. of $F$.

### Questions

(*To be answered whether Part* 2 *has been worked or not.*)

**1.** Draw a diagram of the circuit showing in what parts the current is flowing when the potentiometer is balanced and the resistance box is connected in parallel with $F$, if the box were used. Why does not cell $F$ furnish current to the wire $Ac$?

**2.** Derive the formula used in the computations.

**3.** Why do such frequent readings of $E$ have to be taken?

(*Required only of those working Part* 2.)

**4.** Interpret the curve and find resistance of the cell from it.

**5.** What advantage has the potentiometer over the voltmeter in measuring the e. m. f. of a cell?

## EXPERIMENT 57

## Constant of a Galvanometer. Ammeter Method

Object: *To determine the constant of a galvanometer with a milliammeter.*

By the "constant of the galvanometer" is meant the magnitude of the current which will produce a given deflection. Hence, this constant may have a large number of different values, depending upon the units in which it is expressed and also the distance of the scale from the galvanometer. In this laboratory it is defined as the number of amperes which will produce a deflection of 1 cm. for the particular scale distance that happens to be used with the galvanometer. In general, it is determined by sending a known current through the galvanometer, the deflection produced being noted and the former being divided by the latter. The various methods described in textbooks differ only in the way in which the value of the current is determined. If the galvanometer is of low sensibility, an accurate

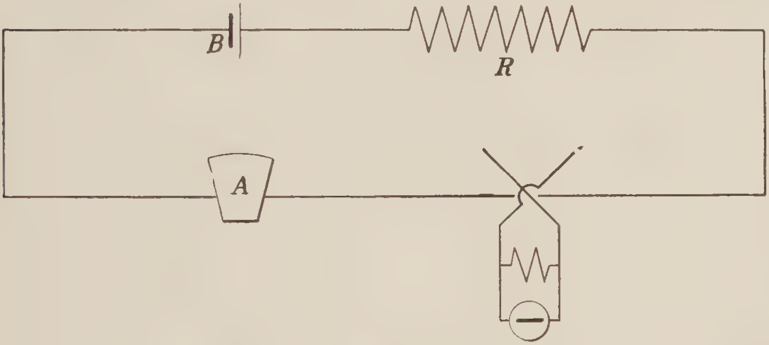


FIG. 43.

ammeter may be used to determine the current flowing. To do this, a storage cell $B$ (Fig. 43), a rheostat $R$, an ammeter $A$, and the galvanometer are all connected in series, and the deflections determined when eight currents are sent through the galvanometer. The ammeter reading and both the direct and the reverse deflections of the galvanometer are to be recorded. The average deflection is to be used in the computations.

**Curve.**—Currents in amperes as abscissas and average deflections as ordinates.

### Questions

1. Make a careful drawing of the galvanometer used, labeling the essential parts and explaining their uses.
2. Interpret the curve.

## EXPERIMENT 58

## Constant of a Galvanometer

OBJECT: *To determine the constant of one of the laboratory wall galvanometers; that is, to determine for each shunt what current through the terminals of the shunt box will produce a 1-cm. deflection.*

### Part I

For shunt 1, lowest sensibility, the method of Exp. 57 may be followed for certain galvanometers (the instructor will designate which ones), using a 0- to 0.5-amp. ammeter. *B* may be either a storage or a *good* dry battery.

### Part II

For the other shunts the circuit is to be arranged as indicated in the diagram. *B* is either a dry cell or a storage battery. In either case its e. m. f. must be carefully measured with a voltmeter. *R* consists either of a slide wire, or of two 1000-ohm resistance boxes *c* and *d*. If two boxes are not available, a single box with a movable plug contact may be used. *R'* is a 10000-ohm box.

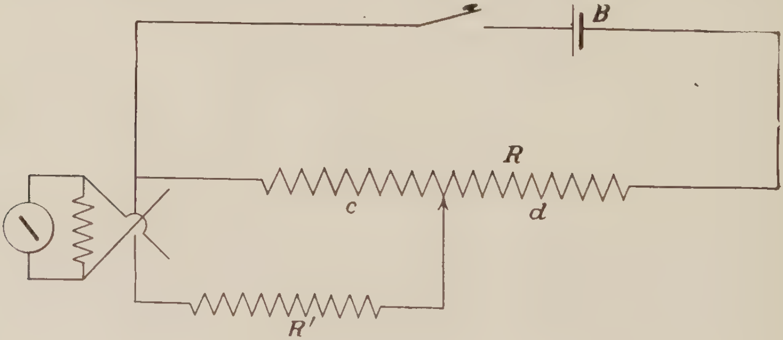


FIG. 44.

Try first the constant for shunt 4 (highest sensibility). Make *c* one-tenth or one one-hundredth of *R* (that is, one-ninth or one ninety-ninth of *d*). This means that one-tenth or one one-hundredth of the full e. m. f. of the battery will be used. Then adjust

$R'$ so that the galvanometer shows a deflection of 10 to 20 cm. $R'$ must be always at least 100 times as large as $c$. Measure the galvanometer resistance (see below) for each shunt.

For shunt 2 and also shunt 3 in some cases it may be possible to do away with $R$ entirely and connect the battery directly in series with $R'$ and the galvanometer.

Make three determinations of the constant for each shunt, with galvanometer deflections of about 10, 15, 20 cm., respectively.

**Galvanometer Resistance.**—This may be found by connecting a resistance $R''$ across the shunt-box terminals and adjusting it until the galvanometer deflection is exactly halved. Then $R_g = R''$. If it is impossible to halve the deflection exactly, $R_g$ may still be found by this proportion:

$$\frac{R_g}{R''} = \frac{D - D'}{D'},$$

where $D$ is the original deflection and $D'$ the deflection when shunted with $R''$. All this presupposes an external resistance $R'$ of 50 or more times $R''$.

### Questions

1. Why must resistance $c$ be only a small fraction of $R'$?
2. Derive the formula

$$\frac{R_g}{R''} = \frac{D - D'}{D'}.$$

3. Why must $R'$ be so much larger than the galvanometer resistance?

## EXPERIMENT 59

## Calibration of Ammeter and Voltmeter

Object: *To calibrate the laboratory ammeters and voltmeters by direct comparison with standard instruments.*

The "standard instruments" of this laboratory are a Model 45 Weston voltmeter with three ranges, and a similar ammeter of two ranges. These instruments are used only for calibration and, as the slightest abuse would spoil them for this purpose, they should be handled with care. In particular, the student is requested to *have the instructor check his set-up* before the current is turned on. Be careful, also, to make no mistake in the matter of the range selected.

Calibrate perhaps four each of the Model 280 Weston voltmeters and ammeters as selected by the instructor. Do this in each case for three points on the scale, giving respectively about one-third, two-thirds, and full-scale deflection of the (Model 280) instrument. Before starting, examine each instrument to see that the pointer is exactly on the zero. If not, turn the zero corrector screw very carefully with the point of a knife. Take the number of each instrument calibrated and file a duplicate of the data with the instructor. Call his special attention to any instrument which is found to be as much as 2 per cent in error. Include in the report a diagram of the connections for this calibration.

**Special Precaution:**

**The standard ammeter is to be used only with one of the two shunts; it will be ruined if directly connected.**

### Questions

**1.** What is the difference in construction between an ammeter and a voltmeter?

**2.** Which would be injured if it were mistaken for the other and connected accordingly?

**3.** The standard ammeter in this case is really a 50-millivolt voltmeter. Explain how, by the use of shunts, it is made to serve as an ammeter.

## EXPERIMENT 60

## Joule's Law. Mechanical Equivalent of Heat

OBJECT: *To determine the mechanical equivalent of heat by the electrical method.*

Energy exists in a variety of forms, and is measured in many different units, but the two units most used are those developed in mechanics and heat. One of the important problems which the physicist has had to solve is the determination of the ratio existing between these two sets. In this experiment, the ratio is to be found using the electric current as a source of heat, hence it is necessary to measure the energy supplied by the current in mechanical units and to determine this same energy in heat units. To do this, a coil of high-resistance wire is immersed in a known quantity of water and an electric current sent through it. As a result, the temperature rises, and from this increase in temperature the amount of energy in calories may be determined. The input of electrical energy in joules can be computed if the difference in potential in volts, the current in amperes, and the time that it flows expressed in seconds are measured.

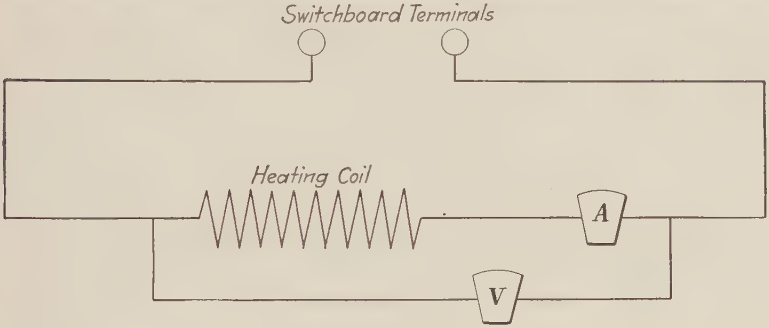


FIG. 45.

In order to obtain reasonably accurate results, the calorimeter is placed in a water jacket, which should be filled with water at room temperature. Since the heating coil gives up its energy rather slowly, better results will be obtained if the final temperature is only slightly above that of the water jacket, as it takes several minutes for the water to reach its true temperature.

In working the experiment, the accompanying directions should be followed:

1. Use about 400 gm. of water cooled 3 to 4°C. below jacket temperature, using the 0 to 40° thermometer to measure this temperature.
2. Connect the heating coil and the ammeter in series with the switchboard terminals, and place a voltmeter in parallel with the coil, as indicated in Fig. 45. After making the connections,

call an instructor to check them, so that valuable apparatus may not be injured. Do not turn on the current until the coil is immersed in water, or it will be burned out.

3. Place the heating coil, without the current flowing, in the water contained in the calorimeter, and, after stirring thoroughly for 2 or 3 min., read the temperature, making two successive readings and taking the mean.

4. Turn on the current and let it flow for exactly 2 min., reading the ammeter and voltmeter every 15 sec.

5. After the current is turned off, stir the water in the calorimeter and read the temperature every 15 sec. until several readings of the same magnitude are obtained. This is the temperature to be taken.

6. The water equivalents of the calorimeter, the stirrer, and the heating coil must be used; the water equivalent of the latter may be taken as 7 gm. Also correct, if necessary from the calibration curves, the ammeter and voltmeter readings.

7. Determine the energy furnished by the electric current in joules and the energy absorbed by the calorimeter and its contents in calories. Then the number of joules divided by the number of calories will give the mechanical equivalent of heat.

8. Make a second run, connecting in series with the heating coil a 100-watt tungsten lamp so as to reduce the current by about one-half. Have the current flow for 10 min. in this case.

9. Before returning the apparatus, empty the calorimeter, but not the water jacket.

10. If the time specified above (2 min.) is not sufficient to heat the water as much above room temperature as it was below at the start, the current may be left on somewhat longer. In any case, however, the time must be carefully measured.

## Questions

**1.** Develop Joule's law from the fundamental definitions of difference of potential and current of electricity.

**2.** Determine *from the data* the relation between the heat developed in a given time and the magnitude of the current.

**3.** Given a 110-volt source of current and two coils of 10 and 100 ohms respectively, calculate the amount of heat developed per minute in each coil: (*a*) when they are connected across this voltage in series, and (*b*) when in parallel. Use for the mechanical equivalent the value obtained.

## EXPERIMENT 61

### Thermoelectricity

OBJECT: *Study of a simple thermocouple.*

1. The student is furnished with a simple thermocouple consisting of two different wires (such as copper and iron, but, as several different combinations will be used, they will simply be known respectively as "wire 1" and "wire 2") joined together at one end. Connect these in circuit with resistance box and galvanometer (shunt 4) and heat the junction in a Bunsen burner. Do this somewhat cautiously, as some wires are easily melted in the blue flame. It is best to turn the burner down low and shut off the air so as to give a white flame. Adjust the resistance to keep the galvanometer deflection on the scale and note this deflection. Double the total resistance (taking account of the galvanometer resistance also) and again note the deflection, heating the couple exactly as before. Would you conclude from this experiment that heating such a junction produces a definite *e. m. f.* or a definite *current?*

2. The student is also given a thermocouple of the same sort, but consisting of *two* junctions—two lengths of wire 1 and one of wire 2, say. Connect this with the galvanometer and the same resistance as before and heat one junction, leaving the other cool. Compare deflections with those previously obtained. What corresponds to the second junction is the first arrangement. Now heat the other junction, leaving the first one cool, and note the magnitude and direction of the current. Then heat both junctions as nearly as possible to the same temperature and note the effect.

3. Calibrate this thermocouple (keeping one junction in ice water), using temperatures of zero, boiling point of water, and one other temperature—perhaps the melting point of tin—plot a curve of temperature and galvanometer deflections. Then test with this thermocouple some intermediate temperature, and compare the value taken from the curve plotted with a thermometer reading of this temperature. Take both direct and reverse galvanometer deflections and use the average throughout this part of the experiment.

**Curve.**—Temperatures as abscissas and galvanometer deflections as ordinates.

### Questions

**1.** What factors determine the *e. m. f.* produced by a given thermocouple? The *current?*

**2.** Is the temperature-deflection curve plotted a straight line? Bearing in mind past reading on the subject, would one expect it to be a straight line?

**3.** Calculate roughly in volts the e. m. f. produced by the thermocouple in the flame. Sketch a system of such couples which would give 1 volt.

**4.** Even if the zero of the galvanometer is carefully adjusted, the direct and reverse readings may not be exactly the same. Suggest a reason for this.

## EXPERIMENT 62

## Electromagnetic Induction

Object: *To study some of the phenomena of electromagnetic induction; in particular to note the e. m. f. induced in a secondary coil by various changes in the relations between it and a current-carrying primary, the earth's field, or a permanent magnet.*

The production of induced currents is one of the most important branches of the study of electricity. The present experiment deals with some of the fundamental principles of this subject. The work outlined is largely qualitative, and care must be taken to make complete records of the observations. As it is important to know the direction of the currents produced in the secondary, it is necessary to find first the direction in which the current flows through the galvanometer when it gives a certain deflection. To determine this direction, connect *one* post of the galvanometer to one terminal of the cell and then connect the other post through the observer's body to the other terminal of the cell. Enough current will flow to produce a deflection. Since the direction of the current furnished by the battery is known, the direction of the current through the galvanometer can be determined. The most satisfactory way to record it is to note at which binding post of the galvanometer the current must enter to produce a deflection in a given direction, *i.e.*, to the black or the red side of the scale.

Two coils will be provided, one wound with coarse wire and the other with fine wire. The former will be called the primary and the other the secondary. Connect the primary in series with a gravity cell and a switch so that the circuit may readily be made or broken. Place the secondary in series with a galvanometer and do not have the two circuits connected in any way while the readings are being made; have them completely independent, but place them side by side and as close to each other as possible. In specifying the direction of the current in the coils, the simplest way is to note whether or not the current flows the same as the hands of a clock or in the opposite direction. The former motion is called clockwise; the latter counterclockwise. Sometimes the terms negative and positive rotations are applied respectively to these two motions. To determine this direction note the deflection of the galvanometer and so find by which binding post the current enters. Then an inspection of the winding of the coil will show whether the current is flowing clockwise or counterclockwise. All that is needed in the primary circuit is to see how the battery is connected. In all this work, care must be taken to view the coils always from the same side.

A list of the observations to be made follows. The *actual magnitude of the first throw obtained in each observation should be recorded on the data sheet. Be sure to record the direction of the current in the primary.* Enter each set of data separately and state what each observation illustrates.

1. (*a*) Move the secondary quickly to a distance of about 150 cm. from the primary. Bring it quickly back to its original position. During these operations, currents should be flowing in the primary.

(*b*) Record direction of currents in primary and secondary for both cases.

2. Repeat observation 1, moving primary, however, instead of secondary.

3. (*a*) Make and break the current in the primary circuit.

(*b*) Record direction of current in primary and secondary for both cases.

4. Leave the primary coil circuit open and be careful to remove any permanent magnets fom the neighborhood of the secondary. Then place the secondary coil so that its axis is vertical and move it upward through about 10 cm. Also revolve the coil through 180 deg. about a horizontal axis passing through the center of the coil. Start with the plane of the winding parallel to the table top. Record deflections. While making all of these observations it is also important to see that the wires leading to the galvanometer do not separate.

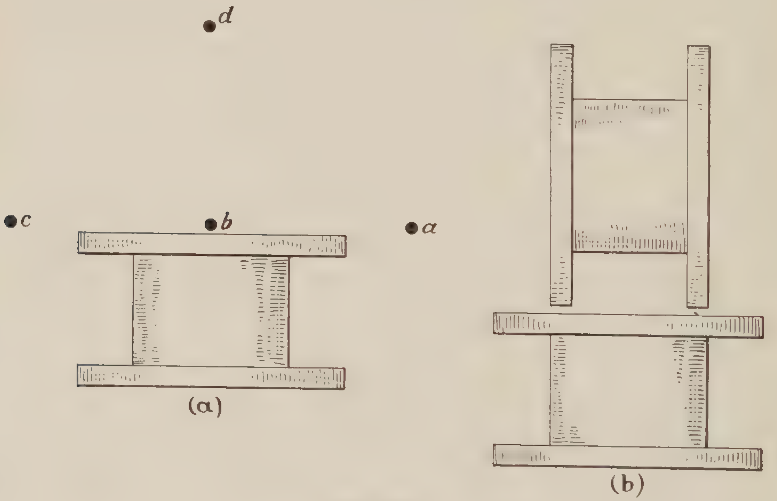


Fig. 46.

5. Determine the throws of the galvanometer when the current is made and broken in the primary for the following distances between the coils: 0, 0.5, 1, 1.5, 2, 3, 4, 5, 10, 20 cm. Measure these distances from the edges of the wooden forms Plot a curve with distances as abscissas and throws as ordinates. Interpret it.

6. Move a magnet from *a* to *b*; *a* to *c*; *d* to *b* (Fig. 46). Note which pole of the magnet was used. The coil indicated in the figure is the secondary. Place the coil so that the plane of its windings is horizontal and note the direction of the current when looking down upon it.

7. Place an iron rod in the holes bored in the primary and the secondary and record the throws when the current is made and broken. Repeat, using a brass rod.

8. Place the coils as shown in Fig. 46*b* and make and break the current in the primary. Do this for several positions of the upper coil. Is it possible to find a position of this coil which will give no deflection of the galvanometer when the circuit is broken?

9. Ask instructor for the small inductor or dynamo model, to be used with this experiment; connect it with the galvanometer, using suitable shunt or external resistance, and note the effect of turning it slowly and also rapidly in each direction.

It will be a great convenience if the data of this experiment are arranged as follows:

| Number of Observation | Direction of Current | Magnitude of Throw |
|---|---|---|
| | Away ________ | ________ |
| | Back ________ | ________ |

## Questions

**1.** Show for observations 1, 3, and 6 that Lenz's law is satisfied; (strictly speaking observation 3 does not come under Lenz's original law but under an extended form of it). For other observations, state what special facts they bring out.

**2.** Can an induced current flow without an induced e. m. f.? Can an induced e . m. f . be present without an induced current appearing?

**3.** State briefly the fundamental principles of electromagnetic induction as illustrated in this experiment.

**4.** Explain the action of the commutator on the dynamo model.

# PART V
# WAVE MOTION AND SOUND

## EXPERIMENT 63

### Lissajou's Figures

Object: *To study the composition of two simple harmonic motions at right angles under varying conditions of amplitude, period, and phase.*

The *Y* pendulum affords one of the easiest ways of compounding two s. h. ms. at right angles. In the present apparatus all variations of period from 1:1 to 2:2 are possible. Reproduce all the figures on page 211 of Carhart's "Physics" or—with the exception of the 3:1 ratio—on page 180 of Duff's "Physics," and study carefully the conditions of period, amplitude, and phase under which they are formed. In particular, choose the ratios 2:1 and 2:3, determining these fairly accurately by timing 50 swings in each direction with a watch. Lastly, slide the clamp as near the top as possible and study the curves formed when the two periods are very nearly equal.

Make some good specimens of each curve on small cards for pasting in the notebook. In doing this, put a little ink on the brush with a fountain-pen filler and hold the card firmly with both hands so that the brush barely touches the surface. When the pendulum is rather high above the table, steady the hands on a box or block of wood. The finer and sharper the line the better.

*Do not cut any of the hairs from the brush.*

### Questions

1. Under what conditions of amplitude, phase, and period will the following figures be produced: (*a*) straight line, (*b*) circle, (*c*) diagonal ellipse, (*d*) ellipse with axes parallel to directions of swing, (*e*) "figure eight"?

## Descriptive Note

**Melde's Apparatus. The Electromagnetically Driven Tuning Fork.**—This apparatus is used to study the wave motion seen in the transverse vibrations of a string. It consists of an electrically driven tuning fork and, attached to it, a string which is set in vibration by the fork. It is the motion of this string which is to be studied; this particular type of wave motion is known as a "standing wave train."

The electromagnet is run by storage batteries—4 volts, or 8 volts if necessary—and some resistance must always be in series. Notice from time to time whether or not the magnet becomes heated. If it does, the current must be shut off and the fork allowed to cool. The smooth running of the apparatus depends upon the proper adjustment of the interrupter. If it is right for one load, it will probably not be in the best position for the others; consequently, it needs more or less continual adjustment, and when the fork stops it can be set in vibration most easily by adjustment of the interrupter. Some idea of the proper place for the interrupter can be obtained by screwing in the disc until the contacts touch and then separating them until a small spark appears. Now moving the disc back and forth through this point will usually start the fork going. *When through with the apparatus, be sure to open the electrical circuit.*

It is sometimes difficult to find the proper tension to make the string give the required form of vibration because several tensions will seem to give equally good results. In that event, the thing to do is to decrease the load gradually until the proper vibration is obtained, then to start with too small a value and increase it until the same condition is reached. An average of the two will give the true value. This does not mean to find the greatest and the smallest loads that will enable the string to vibrate with a given number of segments, but with the nodes poorly defined. What is meant is to find the two extreme loads which will enable the string to vibrate so smoothly that it is impossible to tell which produces the better effect. The tension is most conveniently applied to the string by placing weights on the holder attached to the free end. In calculating, do not forget to include the weight of this holder.

## EXPERIMENT 64

## Vibrating Strings. Melde's Experiment

OBJECT: *To study the relation between tension and wave length in a vibrating string and from such measurements to compute its frequency of vibration.*

A number of factors determine the manner in which a string shall vibrate. The relation which exists among them is represented by the following equation:

$$N = \frac{1}{l}\sqrt{\frac{T}{m}},$$

where $N$ represents the frequency of the string, $T$ the tension in the string, $m$ the mass of the string per unit length, and $l$ the wave length. The experiment consists in studying the relation existing between the tension in the cord and the wave length.

Melde's apparatus will be used. Before starting the experiment read the preceding note, which describes this apparatus. Find the three tensions which will make the string vibrate in three, four, and five segments when the fork is parallel to the axis of the cord. For each tension, find the distance between the nodes. Repeat the observations, placing the fork so that it is perpendicular to the axis of the cord. Compute the frequencies for the six readings, being sure to express all the quantities in c. g. s. units.

**Curves.**—Plot two curves, one for each arrangement, with tensions as abscissas and squares of the wave length as ordinates.

### Questions

**1.** State the relation between frequency, wave-length, and velocity of wave motion. Assuming that the velocity of the wave is equal to $\sqrt{\frac{T}{m}}$ prove that $N = \frac{1}{l}\sqrt{\frac{T}{m}}$.

In what units must $T$ and $m$ be expressed?

**2.** What name is given to the special form of transverse wave motion with which the string moves? How is it produced?

**3.** Explain any differences which may appear in the values of the frequency for the two cases.

**4.** Interpret the curves.

## EXPERIMENT 65

### Vibrating Strings. The Sonometer

OBJECT: *To prove experimentally the laws of the vibrating string; in particular, to determine the frequency of its vibration from measurements of its mass, tension, and wave length and to compare this with the known frequency of the tuning fork with which it is in unison.*

If a vibrating string has its tension or length varied, then the note which it is giving out will change, and it can be shown that there is a definite relation existing between tension, length, and fundamental frequency, which is usually expressed in the form

$$N = \frac{1}{2L}\sqrt{\frac{T}{m}},$$

$N$ being equal to the frequency, $L$ the length, $T$ the tension, and $m$ the mass per unit length. This relationship is to be tested by setting in vibration a string attached to a sounding board and tuning it to three different forks, first by altering the length and then by changing the tension in the wire.

1. Place in the holder attached to the free end of the string a load indicated on the card attached to the instrument, and adjust the length until the string vibrates in unison with the fork of lowest frequency. If some difficulty is encountered in tuning, a small piece of paper bent in the form of a V and placed on the wire near the antinode will greatly help, for, when the string has been adjusted to the proper length, the tuning fork will set it into vibration with sufficient energy to displace the paper. Placing the resonator of the fork in contact with the sounding board will assist in obtaining the effect. Make three settings; then repeat for the other two forks, making the same number of settings for each. Compute the frequency of each fork, being sure to express all quantities in c. g. s. units.

2. Use a constant length of string (in general 45 cm.) for this part of the experiment and vary the tension. Make three determinations for each fork as before and compute $N$.

In each determination, especially for Part 2, rap the wire briskly with a pencil or other light object after changing the tension or moving the fret. Do this at some point between the pulley and the fret at the load end of the sonometer. If this is not done, friction at these places will cause large errors.

**Curves.**—1. Reciprocals of string length as abscissas and frequencies as ordinates (Part 1).

2. Tensions as abscissas and squares of frequencies as ordinates (Part 2).

### Questions

**1.** Assuming that the velocity of the wave in the cord is $\sqrt{\frac{T}{m}}$ derive the formula used in the above computations.

**2.** What name is given to the special form of transverse wave motion with which the string moves? How is it produced?

**3.** Explain why the string gives a louder tone when it is attached to the sounding board.

**4.** Interpret the curves.

## EXPERIMENT 66

## Velocity of Sound. Resonance Tubes

Object: *To determine the velocity of sound in air at ordinary temperatures from measurements of the wave length corresponding to a given frequency, and to compute the velocity at* 0°*C*.

The velocity with which sound is moving in a substance may be determined if the frequency and the length of the wave are known. In this experiment the velocity of sound in air is to be found by using a tuning fork of known frequency to produce a wave whose length $l$ will be measured by means of a resonating column of air.

The air column which is to resonate in unison with the fork is contained in a glass tube, and its length is varied by changing the height of the column of water contained in the lower part of the tube. Projecting from the main tube near its top is a much smaller one, to which is attached a long rubber tube, and, by holding the receiver or funnel in which this ends to the ear, the variations in the intensity of the sound can readily be detected. The point of maximum intensity is most easily found by raising and lowering the water, shifting the indicator at each passage to the point where the sound seems loudest, and continuing until all subsequent passages give the same position of the indicator. Find two points of resonance for the tuning fork and make three determinations of each point.

Now from these resonance points find the length of the air columns corresponding to $\frac{1}{4}l$, $\frac{3}{4}l$, *and,* by subtraction, $\frac{1}{2}l$. As the antinode is not located at the end of the tube, but a short distance out in the air, it is necessary to make a correction. This can be allowed for if 0.6 of the radius of the tube is added to the length. From the three lengths just found, three determinations of the wave length are to be obtained, and, from the average, the velocity of sound is to be found, using the frequency indicated on the fork. The usual way of marking frequencies of tuning forks is to indicate the number of single vibrations, but in the present computation, the number of complete or double vibrations should be used. Thus, 512 v. s. and 256 v. d. represents the number of middle C, but only the latter value should be used.

From the determination made calculate the velocity of sound at 0°C., using the formula

$$V = V_0\sqrt{1 + \frac{t}{273}} = V_0\sqrt{1 + 0.00366\,t}.$$

To save time in computation, make use of the following approximate method of extracting the square roots of numbers which differ slightly from unity, that is, divide the decimal part by 2 and add the quotient to 1. By this method,

$$\sqrt{1.084} = 1 + \frac{0.084}{2} = 1.042,$$

and, since the value obtained by the rigorous method is 1.041, this means an error of less than 0.1 per cent. As the instrumental errors in this experiment are greater than 0.1 per cent, this method can be used in these computations. It should be distinctly borne in mind, however, that this approximation method is allowable only with numbers which are very nearly equal to unity and to no others.

### Questions

**1.** Distinguish between forced and sympathetic vibrations. Which are being dealt with in this case? Which in the sounding board of a piano or the box on which a tuning fork is mounted?

**2.** Explain how the air column vibrates and show why the length of the resonating column must be an odd multiple of $\frac{1}{4}l$. Is there a node or antinode at the closed end? At the open end?

**3.** Calculate the frequency of vibration of the fundamental and first overtone in an open organ pipe 3m. long, the temperature being 28°C. Use the determination made of $V_0$ for this.

## EXPERIMENT 67

## Velocity of Sound. Kundt's Method

Object: *To determine the velocity of sound in two different metals by comparison with the velocity in air.*

One of the commonest ways of determining the velocity of sound is by Kundt's method, which is well adapted for finding it in metals or gases. The apparatus is arranged as shown in Fig. 47. *AB* is a rod of the metal to be experimented upon. It is clamped at its center, and attached at one end is a disc *C* which fits loosely into the glass tube. If the rod is stroked with a piece of cloth or leather covered with a little resin, it can be set in vibration and will produce a loud shrill note. In stroking the rod with the cloth, hold it between the thumb and forefinger; do not grasp it too tightly; and be careful not to pull it out of alignment. The disc attached to the rod should not touch the side of the glass tube but should be free to vibrate. By sliding the glass tube backward and forward, the length of the air column can be varied until the air vibrates in resonance with the rod. This state of affairs is indicated by the fact that the fine cork dust

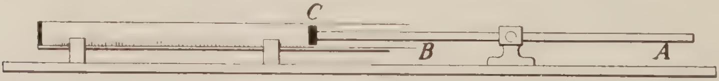


Fig. 47.

contained in the tube is disturbed and made to arrange itself in ridges or striae. Adjust the tube so as to get the largest number of nodes and then locate the position of each node by means of a meter stick placed beneath or beside the glass tube. Redistribute the cork dust, displace the meter stick a trifle, and make another determination. Three such runs should be made.

As these striae mark out very distinctly the position of the nodes and the antinodes, the wave length in air of the note emitted by the rod can be measured. From each set of data, compute the distance between nodes following the method of Par. 10 of the Introduction. From the wave length determine the frequency of the note, using that value for the velocity of sound in air which corresponds to the particular temperature of the experiment. The expression which gives this velocity is

$$V = 33200\sqrt{1 + 0.00366t}.$$

For an approximate method of extracting square roots, see the preceding experiment. Now compute the velocity of sound in the metal. This can be done since the rod is vibrating in unison with the air column and the wave length of the note in the metal can be readily determined from the length of the rod. Two pieces of apparatus will be supplied which have rods of different metals, and complete sets of data are to be taken with both.

**Optional.**—Though the wave motion in the rod is here longitudinal, it is convenient to represent it diagrammatically by a transverse diagram. The student should draw such a diagram for the fundamental mode of vibration and also draw the first three overtones. Indicate on the drawing the wave length in each case and show to the instructor for his approval. If the drawings are correct, they will show that the first and third overtones have an antinode at the middle of the rod, and, since the rod is clamped here, such a mode of vibration will be impossible. The second overtone, however, has a node here and may be produced by stroking the rod not near the end (which will give the fundamental), but about one-sixth of the rod length from the clamp. (Why would stroking here emphasize this overtone, while stroking at one-third the rod length from the clamp would surely not give it?) The first and third harmonics can be obtained only by changing the position of the clamp so that it falls on a node for the particular wave length desired—such nodal points can be determined from the diagrams made. In the case of the brass rod, the students should obtain the second overtone by stroking the rod at the proper place and, by changing the position of the clamp, get the first and third overtones. While it is sometimes difficult to obtain the dust piles, they may usually be obtained for the first and second overtones, and the student should measure these and use them to compute the velocity of sound. If the fundamental is called "Do" what will be the first, second, and third overtone?

*Because of the disagreeable character of the sound, do not stroke the rod more than is necessary to secure well-defined nodes. See that the doors of the room are closed.*

### Questions

**1.** Describe in detail the wave motion in the rod and the air column.

**2.** Explain why the frequency of the vibrating rod is the same as that of the air column.

**3.** Try one experiment with the end of the tube *open*. How does this alter conditions?

*Optional*

**4.** Is there an antinode or a node in the vibrating air column at $C$? Explain with diagram.

# PART VI

# LIGHT

## Descriptive Note

**Photometry.**—The Bunsen photometer is employed in this laboratory for comparing the intensities of different lights, such, for instance, as two incandescent lamps. The screen used consists of a piece of plain paper with a star-shaped portion rendered translucent by impregnating with paraffin. If the lights are of the same color, the star will practically disappear if viewed from one side; but, when the other side is examined, the star will usually be very distinct. It is therefore impossible to make the star disappear on both sides at the same time; the aim is to place the screen so that the contrast between the star and the untreated part seems to be the same on the two sides. As it takes considerable practice to set the screen consistently, and a beginner usually has trouble in doing it, it will be found necessary for each observer to make five independent settings for each determination and to take the average, if results of any value are to be obtained. It is advisable for one observer to make a complete set of readings while his partner makes the necessary adjustments and reads the electrical instruments, if that is required. This allows the person setting the photometer to shield his eyes from all light except that coming from the screen, thus enabling him to make more accurate settings.

The distance between the lamps will vary with the particular photometer used but will be specified on the card attached to each instrument. At the beginning of the experiment this distance should be checked to see if the lamps are correctly placed. Also care should be taken to have them at the same height as the screen and so placed that it is evenly illuminated. This last adjustment is best made by watching the screen carefully as the box is rotated, and placing the box so that the untreated part of the screen is uniformly illuminated.

There are several standard lamps, *e.g.*, the amyl acetate burning standard, available at the present time, but they are troublesome to use and add difficulties in making accurate settings on account of differences in color. Accordingly, no attempt will be made to employ them, but, instead, an incandescent lamp of known intensity will be used. The value of its intensity will be marked on the bulb.

## EXPERIMENT 68

## Vertical Distribution of Light from an Incandescent Lamp

OBJECT: *To measure the intensity of light at a given distance from an incandescent lamp for various points in a vertical plane passing through the center of the filament.*

From the data obtained in this experiment such questions as the following may be answered: In what position should an incandescent lamp be placed to give the greatest amount of light directly beneath it? Before starting this experiment the student should read carefully the preceding note on the photometer and follow the directions given there. The lamp is mounted on a special holder which allows it to rotate about a horizontal axis and the lamp should be placed so that this axis passes through the center of the lighted filament. The holder is of such a form that the lamp may be held at various angles to the vertical and the intensities are to be determined for all the positions which can be obtained on the apparatus. The only exception is when the base of the lamp shuts off all light from the photometer screen, as it does in one case. This means that determinations of intensity for 11 positions of the lamp are to be made. The standard lamp will burn with the intensity marked on it only when there is a certain difference of potential at its terminals, but, as both lamps will be attached to the same circuit, it will be assumed that they will vary at the same rate under changing electrical conditions: hence, no attention will be paid to this variation. In making the computations, only three significant figures are to be retained and the table of squares at the end of the manual will greatly lighten the work.

**Curves.**—Use polar coordinates, *i.e.*, draw lines radiating from a common point, whose angles with the vertical represent the various positions of the lamp and whose lengths are proportional to the intensities. Connect the ends of these lines by a smooth curve.

### Questions

**1.** Show why the intensity of illumination should vary inversely as the square of the distance.

**2.** Explain completely why the intensities of the two sources vary directly as the squares of the distances from the photometric screen.

**3.** Compare the observations made by the two observers to see if there is more or less regular disagreement between them.

**4.** Interpret the curve.

## EXPERIMENT 69

## Efficiency of Incandescent Lamps

OBJECT: *To study the relation between the light energy emitted by an incandescent lamp and its consumption of electrical power.*

Two similar 25-watt tungsten lamps will be used, one of which will be regarded as a standard and kept burning at constant voltage, while in the other the voltage, and consequently the power consumption, will be allowed to vary. To determine the power consumed in this, which will be called the test lamp, an ammeter is connected in series with the lamp and a voltmeter having a range of 0 to 120 volts is placed in parallel. From the readings of these two instruments the power consumed is to be computed by the usual expression. After making the connections have an instructor check the circuit before closing the switch.

The intensity of the light is to be measured by the Bunsen photometer; before starting the experiment the note before Exp. 68 should be read carefully. The directions given in this note should be followed and five settings of the screen made by each student for each particular current in the test lamp. Ten currents should be chosen for this lamp, varying from the highest obtainable for the given voltage down to one which will reduce the brilliancy of the lamp to about one-tenth its normal amount. The intensity of the standard lamp will naturally vary with any change of potential due, for instance, to a variation in the e. m. f. furnished by the dynamo. This is, however, supposed to be nearly constant and, if any such variation appears, the student should at once consult the instructor.

**Curves.**—Plot two curves on different sheets:

1. Power consumption of test lamp as abscissas and light emission as ordinates.

2. Voltage on test lamp as abscissas and light emission as ordinates. In each case call the emission of the standard lamp unity.

### Questions

**1.** Discuss the results obtained in this experiment. What does the curve of power consumption and light emission allow one to conclude as to the most efficient conditions under which to run such a lamp? What would limit the voltage at which it would be profitable to run such a lamp?

## Descriptive Note

**Parallax.**—By parallax is meant the apparent displacement of one body with respect to another when the position of the observer is changed. A good example may be observed from a railway car window, where objects in the foreground appear to be rushing backwards at a much greater rate than those farther away. A very simple experiment in this connection, which the student should try, is to hold two pencils at arms length, in line with one eye but one a few inches behind the other. As the head is moved from side to side, the pencils apparently shift with respect to each other and this shift will disappear only when they are brought together.

The method of parallax is, accordingly, a very useful way of deciding when two objects, nearly in line, are the same distance away. The method has already been made use of in one or two connections, in particular, in determining when the cross-hairs in the reticle of the micrometer microscope are in the plane of the image. There are also numerous other applications, some of which will be found in the following experiment.

## EXPERIMENT 70

## The Concave Mirror

OBJECT: *To study the laws of reflection of the concave mirror and to determine its radius of curvature by three independent methods.*

The concave mirror is rapidly becoming one of the most important optical instruments. While not used to any such extent as the lens in small pieces of apparatus, it is continually attaining more and more importance in the astronomical field, for the largest telescopes in the world, *e.g.*, the one at Mount Wilson, Cal., of 101-in. aperture, make use of huge concave glass mirrors with silvered surfaces.

The apparatus for this experiment consists of a type of optical bench along which slide in guides a special incandescent lamp *A* (Fig. 48) used as an object and a screen *B* on which the image is to be focused. The lamp bulb is frosted and painted black, save for a small part of the surface on which is drawn the figure used as an object. The spherical concave mirror is mounted in the receptacle at the end of the bench and is in permanent adjustment. It is of glass with a silvered surface, which would be injured by the slightest touch. Therefore, *do not touch the mirror* either with the finger or with any other object. Note that the scale on which the object distances and image distances are to be read is inset at the end so that it measures directly from the surface of the mirror. The object distance is given on this scale by the reading at the indicator *I*. In addition to the above, a telescope will be required in Part II of this experiment.

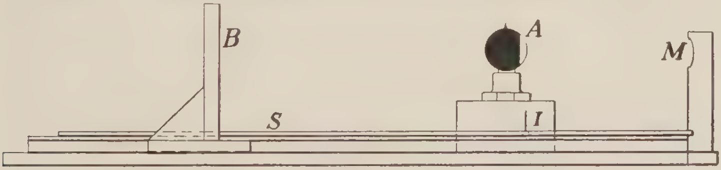


FIG. 48.

### Part I

The images formed by a concave mirror, for various positions of the object, have different characteristics which are to be noted and studied, while measurements are to be made of the corresponding object and image distances from the mirror. The data should be arranged according to the form indicated below. Eight positions of the object will be used and the corresponding images located. The first object distance should be as great as possible, *i.e.*, about 100 cm. Various shorter distances—*e.g.*, 80, 60, 45, 30 cm., etc.—should then be tried until at the sixth the image is cast on the wall, the screen being removed and the corresponding distance measured. For the last two distances, say 15 and 8 cm., the lamp is inside of the principal focus, in which case the image will, of course, be virtual. To locate its position, the method of parallax will be used (read preceding note on parallax). Have someone hold a pencil behind the mirror and then determine, by moving the head from side to side, when the image and the pencil appear to be at the same distance on the other side of the mirror. Use as an object in this case not the lamp, but the brass strip beside it. The image distance, *i.e.*, the distance the pencil is behind the mirror, can then be measured, but it will be found impossible to make these determinations with anything like the accuracy of those for the real image.

In computing the radius of curvature of the mirror use the familiar equation

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r},$$

where $u$ and $v$ are the corresponding object and image distances respectively, and $r$ the radius of curvature. Compute the radius of curvature for each of the eight positions and tabulate in the third column as indicated below. In determining each image distance of the above eight, *make three settings* in succession and use the average.

| Object distance | Image distance | Computed radius of curvature | Image is | | |
|---|---|---|---|---|---|
| | | | Real or virtual | Erect or inverted | Diminished or enlarged |
| | | | | | |

## Part II

If a bright object is placed at the principal focus of a concave mirror, the light (from each point) will be reflected in a parallel beam, *i.e.*, a plane wave. If some means of detecting parallel light is at hand, the placing of the object at the principal focal distance from the mirror may be very accurately done. One of the most convenient means of testing for parallel light is to use a telescope which has been focused on a distant object, for then it will form a sharp image only when parallel light strikes its objective. The telescope is to be focused through an *open window* on the most distant object obtainable. It is necessary to do this focusing through an open window because ordinary window glass has such poor optical surfaces that a sharp image cannot be obtained through it. Then, *without the focus being disturbed,* the telescope is mounted in its holder on the tripod and pointed at the mirror. After a little adjusting it will be found that there is one position of the object—use the sharp edge of the brass strip as the object—for which a sharp image is obtained. This object distance is then the principal focal length. Five determinations are to be made and the radius of curvature computed.

One of the characteristics of the concave mirror is that it shows considerable astigmatism, *i.e.*, a tendency for the images of points to draw out into lines, if both object and image do not lie very near the axis. This is the reason for using the brass strip instead of the lamp in the above measurements. The astigmatism will be noticed at once if the lamp is viewed in this way.

## Part III

When a pencil point is placed in front of a concave mirror at certain distances, its inverted real image can readily be seen and if the observer moves his head from side to side the object and its image will, in general, show parallax. When the object is shifted until this parallax no longer exists, the object and image distances are obviously identical and both object and image are at the center of curvature of the mirror.

In this part of the experiment the electric light will not be used as an object but rather a card held flat against the screen with one corner projecting slightly. The observer should look toward the mirror and focus his eyes not on the mirror itself, but on the object, *i.e.*, the corner of the card. When the adjustment is correct, the point of the card apparently touches its image. Five determinations of the radius of curvature are to be made in this way.

**Curve.**—Object distances as abscissas; image distances as ordinates.

### Questions

**1.** Draw a vertical line as nearly as can be estimated asymptotic (*i.e.*, tangent at an infinitely great image distance) to the curve. What is the physical significance of the abscissa of this line?

**2.** Are there any general relationships existing among columns 4, 5, and 6 in the table for Part I? Thus, if the image is virtual, is it always inverted and reduced or is the reverse true?

**3.** Explain why the method of Part II gives the focal length. Is it necessary to know the distance of the telescope from the mirror?

**4.** Why is the position of no parallax in Part III the center of curvature of the mirror? Deduce from the mirror equation the relation between radius of curvature and focal length.

**5.** Make two constructions of the image formation, one for a real and the other for a virtual image.

*Optional*

**6.** Note that the curve shows no negative object distance. What would such mean? Is it experimentally realizable?

**7.** Explain the "Magic Bouquet" experiment on the basis of the observations in Part III.

## EXPERIMENT 71

## The Eye as an Optical Instrument

Object: *A study of the Eye as an Optical Instrument, with the aid of a Model.*

A simple but profitable study of the eye and some of its defects may be made with the aid of a model (an adaptation of the one devised by Kuehne). This (Fig. 49) is a small brass tank about 18 cm. long and 12 deep, shaped roughly like an eyeball. It is to be filled three-fourths full of tap water, which takes the place of the aqueous and vitreous humors. The "retina" is held by pins in any one of three positions—$R_o$ for the normal eye, $R_P$ for the farsighted eye (short eyeball), and $R_M$ for the nearsighted (long eyeball). The "cornea" $C$ is fixed, but the "crystalline lens" $L$ is removable. Various lenses may be placed in the guides *a*, *b*, *c*, *d*. It will be found a little better to place all the lenses so that they are on the side of their brass mountings *away from the cornea*, as indicated in the figure.

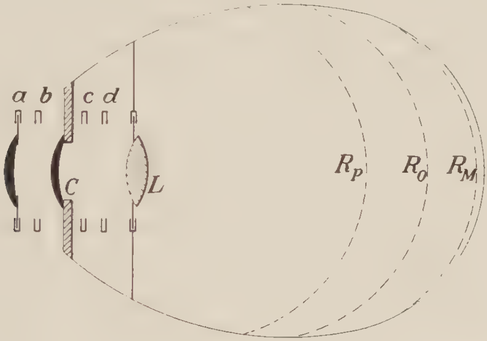


FIG. 49.

**1. Accommodation.**—Set up the model on a table facing a window or other bright object between 4 and 5 m. away. If the retina is at $R_o$ (normal eye) and the converging lens 1 (stamped on mounting) is put in position as the crystalline lens, the object should be in focus. Note the character of the image, its inverted position, what happens when it falls on the "blind spot," etc. Next, use as an object the lamp box with an eight-armed cross and place it just 35 cm. from the cornea. The image will be very indistinct until lens 1 is replaced by the stronger lens 2. This corresponds to the process of accommodation or focusing of the real eye, which is done by changing the curvature of the crystalline lens by means of a muscle attached to it.

Move the model until the image is indistinct and then note the sharpening effect produced by using the diaphragm with small opening, thus decreasing the size of the pupil.

**2. Near- and Farsightedness.**—Using as the crystalline lens either 1 or 2, adjust object distance to give a sharp image on the retina at $R_o$. (Unless the day is very bright use lens 2 and lamp box at 35 cm.) Then produce myopia or nearsightedness by placing the retina at $R_M$. Examine the correcting lenses 3 and 4 carefully and decide the type, *i.e.*, converging or diverging, necessary to correct this defect. Test the conclusion reached by placing correcting lens in front of eye and see if it sharpens the image. Repeat for hypermetropic or farsighted eye.

**3. Astigmatism.**—Using the stronger crystalline lens and the retina for the normal eye, insert *behind* the cornea the cylindrical concave lens 5, thereby producing astigmatism. (In the human eye astigmatism is generally due to a slight cylindrical curvature of the cornea. In the present model a change of cornea would be the logical way of producing it, but the insertion of an additional lens accomplishes the same purpose with less trouble.) By turning 5 a little, one way or the other, one line alone of the cross can be made sharp, the others being quite indistinct. Now place in front of the cornea correcting lens 6, which is convex cylindrical, and turn until the image is again sharp.

**4. Vision without Crystalline Lens.**—In some diseases of the eye (*e.g.*, cataract) the crystalline lens must be removed. Vision is still possible, however, by the use of a suitable correcting lens. Remove the lens in the present case and place in front of cornea lens 1. A clear image will be formed for a very near object.

**5. Compound Defects.**—Astigmatism is frequently accompanied by near- or farsightedness. Combine the tests 2 and 3, using both cylindrical and spherical correcting lenses at the same time in front of the cornea. In actual practice the two correcting lenses are, of course, combined in a single compound lens.

**6. Action of Simple Magnifier.**—Use crystalline lens 2 and lamp box at 35 cm. as object—normal eye. Now introduce lens 1 as a simple magnifier in front of cornea. For a clear image the eye will now have to be moved nearer the object until its distance is only about a third of its former value: the image will accordingly be about three times as large. This shows that the usefulness of the magnifying glass is *merely to enable one to get the eye close to the object* and still see it distinctly.

Note in this connection that, for close work, the near-sighted eye has an advantage over the normal eye. With crystalline lens 2 and lamp box at 35 cm., retina at $R_o$, note approximate size of image. Then place retina at $R_M$ and move up until image is again sharp: it will, of course, be considerably larger. This would indicate that a near-sighted jeweler, say, could do with naked eye a great deal of work for which a man with normal vision would require a glass.

At the conclusion of the experiment, empty the eye model. Clean the six lenses with a little absorbent cotton and replace them in the box.

### Questions

**1.** Hold near the eye the converging and diverging lenses 3 and 4, and move them from side to side. In which case do objects move *with* the lens and which against? How is the effect altered in the cylindrical lens?

**2.** Using one's own or a friend's glasses, test as above and decide the character of the eye defects they are intended to correct. Note particularly if the two lenses are the same and, if not, which is the stronger. Also look for astigmatism.

**3.** Lens 3 has a focal length of 57.2 cm., or a "power" of $\frac{100}{57.2} = 1.75$ "diopters." Lens 4 is of $-66.7$ cm. focus, or of $\frac{100}{-66.7} = -1.5$ diopters. Perhaps one's own or a friend's eyeglasses can be neutralized with one of these and its power thereby determined. Lenses 1 and 2 have powers respectively of +7 and +20 diopters; calculate their focal lengths. How are these changed by immersing in water?

**4.** With eyeglasses, would any harm be done if a spherical lens should become turned in the frame? An astigmatic lens? Why?

## EXPERIMENT 72

## Index of Refraction. Spectrometer

Object: *To study the spectrometer and with it to determine the angle of a prism and its minimum deviation for sodium light; also, from this data, to calculate its index of refraction.*

If light passes through a prism, it will not emerge parallel to its former path but will be inclined to it; the angle thus formed is usually spoken of as the angle of deviation. By varying the direction of the incident ray, it will be found that the angle of deviation varies in magnitude, but that for one particular angle of incidence it has a minimum value. When this critical value has been obtained, it can be proved (see any physics text) that

$$n = \frac{\sin \frac{1}{2}(a + d)}{\sin \frac{1}{2}a},$$

$a$ representing the angle of the prism, $d$ the angle of least deviation, and $n$ the refractive index of the material of which the prism is composed. Since these angles are easily measured, the refractive index may be readily determined.

In order to measure the angles $a$ and $d$, a spectrometer is used. Figure 50 shows a form of this instrument similar except in minor details to the one used in the laboratory. An important part of the spectrometer is the graduated circle $A$, which is capable of rotation about a vertical axis. Attached to the axis carrying this circle is the prism table $B$, which has a special set of leveling screws and can also be adjusted vertically. The telescope $T$ is so mounted that it also revolves about the same axis and has rigidly attached to it two verniers 180 deg. apart, by which the graduated circle is read. The collimator $C$ which carries the slit $S$ completes the essential parts.

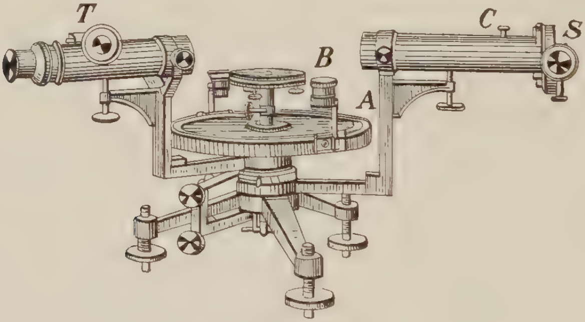


FIG. 50.

## Part I

**Adjustments.**—Before accurate readings may be obtained, the instrument must be thoroughly understood and the use of each milled head and set screw known. First focus the telescope on a distant object *through an open window*, at the same time adjusting the eyepiece relative to the cross-hairs so that there is no parallax, *i.e.*, so that the image and the cross-hairs do not move with respect to each other when the eye is shifted. When the telescope has once been focused, do not alter it. Now, turn it so that the collimator is in line and see that both tubes are, as nearly as the eye can judge, horizontal, *i.e.*, perpendicular to the axis of rotation. To make this adjustment accurately is a somewhat difficult process, but the eye will serve very well for present purposes. Adjust the draw tube of the collimator until a sharp image of the slit is formed. Lastly, see that the prism table is at the proper height, but do not alter the adjustment of its leveling screws.

## Part II

**Determination of a.**—Place the prism so that the refracting edge, *i.e.*, the one opposite the ground face, is near the center of the table and pointing toward the collimator as shown in

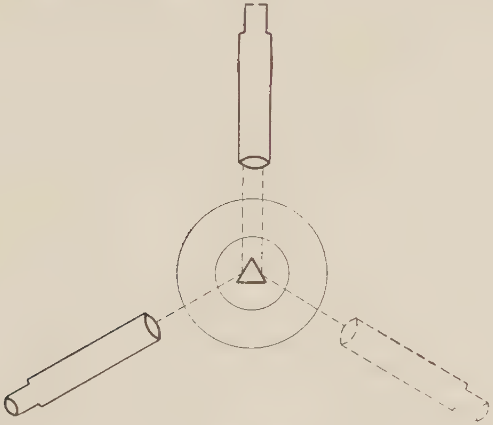


Fig. 51.

Fig. 51. Illuminate the collimator slit with any convenient source of light, such as an incandescent lamp or a sodium flame, such as will be used in Part III. Move the telescope until an image of the slit reflected from one surface of the prism is in the field, and then turn it until the reflection from the other face can be seen. Set the cross-hairs on one edge of the slit, *always using the same edge*, and read both verniers. Do this for both images and the angle $a$ will be one-half the difference between the two readings on the same vernier. Do not make the mistake of subtracting the reading of one vernier from that of the other but treat each set separately. Turn the prism table so as to change the readings and *make two more determinations of this same angle.*

### Part III

**Determination of d.**—To find $d$, the angle of minimum deviation, a sodium flame, which furnishes substantially monochromatic light, will be used. This is most easily produced by placing a small crystal of rock salt on the grid of a Meeker Bunsen burner. The best flame is obtained when only a single small piece of salt is used. Do not let the flame come too near the collimator slit. Place the prism carefully on the table, with refracting face adjacent to the telescope perpendicular to the line of sight. The correct position of the prism is shown in Fig. 52, where the light is shown passing through the prism parallel to the ground surface. An inspection of the apparatus will show whether the ray is turned to the right or to the left, and, if the telescope is rotated in the proper direction, the slit image can be found. This gives the deviated ray, but is not necessarily at the angle of minimum deviation. To find this angle, rotate the prism table, either with or without the graduated circle, until a point is reached where the image of the slit will stop and retrace its path even when the rotation of the table is not reversed. If, in locating this turning point, the image of the slit tends to move out of the field of the telescope, the telescope must, of course, follow it. This turning point is the position of minimum

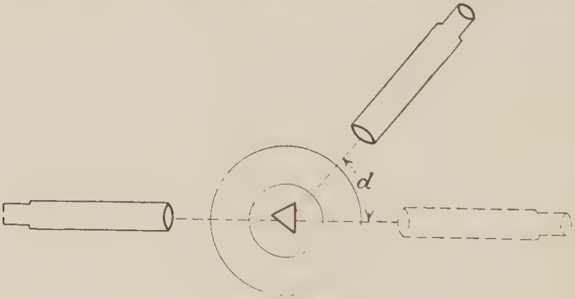


Fig. 52.

deviation and the table should be set as near to it as possible. Then move the telescope until the cross-hairs coincide with the slit and read both verniers. Remove the prism, line up the telescope and collimator, and read both verniers again. The difference between the settings is the angle of minimum deviation. Make *three more determinations of d*, arranging it so that two positions of deviation are to the right of the line of sight and two to the left. Compute the index of refraction of the prism.

Finally, illuminate the slit with white light and note the spectrum formed so as to be able to answer question 3 below.

Tabulate all measurements of Parts II and III.

### Questions

**1.** Show that the angle through which the telescope moves is twice the angle of the prism.

**2.** Explain why monochromatic light is needed when the angle of minimum deviation is determined.

**3.** Explain the spectrum formed when white light is used. Which color is deviated most? For which color, red or blue, has glass the higher refractive index?

**4.** Observe the mercury spectrum in the museum. Describe and explain what is seen.

## EXPERIMENT 73

## Focal Length of a Convex Lens

OBJECT: *To determine the focal length of a convex lens by three methods.*

Two types of optical bench are available for this experiment, one of which is that primarily intended for Exp. 70 It is immaterial which is used.

1. Place a plane mirror immediately back of the lens, so that the light is again reflected through the lens onto a screen. When working with the optical bench of Exp. 70, use as a screen a card held against the lamp, so as just to come to the edge of the object figure. Now move the lens until a sharp image is formed; the distance of the object or screen from the lens then represents the principal focal length. *Make five determinations.*

2. Adjust the object and lens so that a distinct image is formed, using as a screen the white block at the end which contains the concave mirror. Choose four positions of the lens and object for which a sharp image is cast on this surface, two of them being such as to give enlarged images and two of them diminished images. For each of these positions *make three settings* of the object and read carefully the distance from screen to lens and from lens to lamp. In reading these distances be sure to correct for the fraction of the centimeter which the rule is inset in the block at the end. The focal length of the lens can then be computed by the use of the expression[1]

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f},$$

where $u$ represents the object distance and $v$ the image distance. *Make four such determinations*, one for each position of the object and lens.

3. Place the light so that the object is just 100 cm. from the screen, *i.e.*, a few millimeters beyond the end of the meter stick. (If the larger apparatus is used, this distance may be made somewhat larger. When using this apparatus it is also wise to reverse the lens for alternate measurements.) There will then be found two positions of the lens for which a distinct image will be formed, one giving an enlarged and the other a diminished image. If $L$ is the distance between the object and the screen, and $a$ the distance between the two positions of the lens, then

$$f = \frac{L^2 - a^2}{4L}.$$

This expression can be derived from the formula given in 2 if $L$ and $a$ are expressed in terms of $u$ and $v$, taking into account the simple relation which exists between the image and object distances of the first position and the second. Make three determinations of $f$, using three values of $L$, *viz.*, 100, 95, and 90 cm. (These distances may be somewhat greater if the larger

---

[1] *This law is frequently written*

$$\frac{1}{u} - \frac{1}{v} = \frac{1}{f}.$$

The student may use the formula to which he is accustomed, but in the latter case careful attention must be paid to sign.

apparatus is used.) For each of these *make three settings* of the lens position.

Tabulate the results of methods 1, 2, and 3 for comparison purposes.

### Questions

1. Show that the first method gives the focal length.
2. Derive the formula used in method 3.
3. Describe the changes in the character of the image as the object moves from infinity up to the lens.

## EXPERIMENT 74

## Optical Instruments

OBJECT: *To build up and study the optical system and method of image formation in the case of the telescope, the opera glass, and the compound microscope; also to determine the magnifying power of a simple telescope.*

The apparatus consists of four simple lenses, three converging and one diverging, with their holders, a diaphragm with aperture and cross-wires, and an optical bench on which to mount them. The apparatus is to be located across the room from the large vertical scale.

The first step is to determine approximately the focal lengths of the various converging lenses. This is to be done by forming an image on a card of some distant object, *e.g.*, a building or a distant light, or—best of all—the sun. These measurements will be of use in all of the parts which follow.

### Part I—The Telescope

Build up a rudimentary telescope, using the lens of longest focal length as objective and one of the short ones as eyepiece. Use an electric light across the room as an object. Find, by using a card as screen, where the real image is formed by the objective and mount the eyepiece lens at about its focal length behind this point. Focus to get a clear image. Next mount the diaphragm in the plane of the real image and adjust until both image and cross-hairs are in sharp focus and show no parallax.

Also form an *erecting* or terrestial telescope by the use of the second short-focus lens. The eyepiece lens is to be moved back and the other lens introduced, so that it forms a *real erect image of the real inverted image* from the objective. Focus on this image with the eyepiece lens. Repeat, putting the diaphragm between the two short-focus lenses.

Make a drawing, approximately to scale, of the image formation in each of these two optical systems.

Special care must be taken, both in this work and in that which follows, to have the various optical parts at the same height and lined up. The lenses must also be perpendicular to the optical axis or line through their centers. Most of the difficulties encountered in this experiment may be traced to a failure to observe these precautions. Moreover, if the eye is moved too far from the optical axis in testing for parallax, imperfections of the lens may cause a spurious movement of the cross-hairs. Accordingly, in such tests the eye should be moved only slightly from its central position.

## Part II—Magnifying Power of Telescope

By magnifying power is meant the ratio of the angle which the image subtends at the eye to that subtended by the object. By making certain approximations, it can be shown that this equals $\frac{V}{u}$ where $V$ is the distance of the image formed by the objective from that lens and $u$ the distance of the eyepiece from the same image.

Build up a simple inverting telescope as described in Part I, using the diaphragm, and focus on the large vertical scale, adjusting until the image and cross-hairs show no parallax. If, now, both eyes are kept open, the left, say, looking directly at the scale, while the right views it through the telescope, the magnified image of the diaphragm opening and scale (Fig. 54) will be superposed on the unmagnified scale (Fig. 53).

The ratio of the number of divisions seen in each case gives the magnifying power at once. For if, as in the present example, two scale divisions when magnified subtend the same angle as eight unmagnified, the apparent increase in size of each division is obviously fourfold.

One special precaution must be mentioned: If the head is moved up and down it will generally be found that the scale as seen with the naked eye and the image in the telescope will be displaced with respect to each other. Explain this parallax (see question 3). It should be eliminated by moving the eyepiece a little farther away from the objective.

Make three determinations of magnifying power and compare with the computed value of $\frac{V}{u}\cdot$

After $u$ and $V$ have been determined, it will be more convenient in many cases to dispense with the diaphragm entirely in the foregoing measurements and get the magnifying power directly from the relative apparent sizes of the two superposed scales. In everyday practice a brick wall serves very well in place of such a scale.

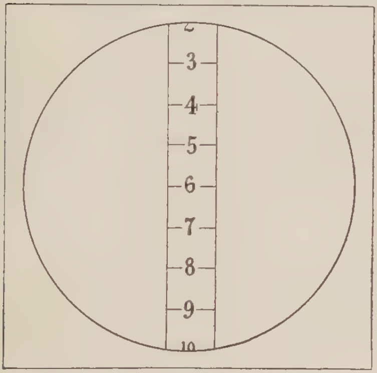


FIG. 53.

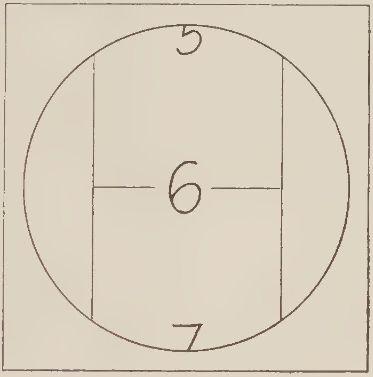


FIG. 54.

## Part III—Opera Glass and Microscope

An opera glass or Galilean telescope and a compound microscope in its most rudimentary form are to be built up, following the descriptions in any physics text. Determine the position of the real image in each case and make approximate scale drawings of the image formation. Measure roughly the magnifying power of the opera glass.

### Questions

**1.** Derive the expression used for the magnification.

**2.** Why should the magnification of a telescope become somewhat greater when close to the object?

**3.** Explain why there is parallax between object and image when the telescope forms the most distinct image. Why does moving the eyepiece out eliminate this?

**4.** Why cannot cross-hairs be used with the Galilean telescope?

## EXPERIMENT 75

## The Diffraction Grating

OBJECT: *To become acquainted with the diffraction grating and to measure the wave length of sodium light.*

One of the remarkable achievements of experimental physics has been the measurement of the wave lengths of light, for an accuracy of the order of a billionth of a centimeter is involved in the most exact determinations. This is accomplished by an indirect method and the diffraction grating is usually employed. This instrument will be used in the present experiment, but, of course, no attempt will be made to secure as accurate a determination as mentioned above; an accuracy of at least one-millionth of a centimeter, however, will be expected.

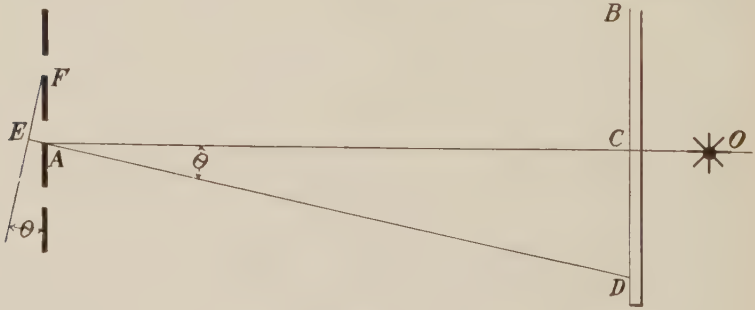


FIG. 55.

The grating to be used is a collodion-film reproduction of one of the famous Rowland gratings and is of the transmission type. Figure 55 shows the general arrangement of the apparatus. *AF* represents the grating, much magnified with respect to the rest of the apparatus, *BD* the scale on which the position of the image is to be located, and *O* a sodium flame. Use only a single small piece of salt with this flame.

From the theory of diffraction the (nearly) plane waves from the source *O*, after passing the grating, spread out from the transparent spaces between the rulings as a series of Huyghens wavelets. One way in which these can recombine is so as to form waves parallel to the original ones—a statement which is at once proved by the fact that the source *O* appears unchanged in position when viewed through the grating. But this is not the only way in which recombination can take place, for the wavelet just starting from (the lower edge of) the slit *F* is exactly in phase with the one just a wave length *l* ahead of it from *A*, the one two waves ahead from the next slit, etc., and a new wave *EF* may be formed from these wavelets all in the same phase. The eye, near *E*, receiving these waves accordingly sees an image of the flame at *D* and another near *B*.

From similar triangles

$$\frac{AE}{AF} = \frac{CD}{AD}$$

or

$$AE = l = AF\,\frac{CD}{AD}.$$

*AF* is the distance from the edge of one groove to the edge of the next, and this can be determined from the number of lines ruled per centimeter. This number is marked on the grating, usually expressed, however, in terms of the number per inch. The distance *AD* is difficult to determine, so it is more satisfactory to measure *AC* and *CD* and to find *AD* either from the trigonometric functions or the relation existing among the sides of a right triangle. Since the source of light is quite near to the observer, the wave front *EF* is not a true plane, but the pupil of the eye admits such a small portion of this surface that it can be assumed to be plane without any appreciable error being introduced.

The points of similarity, as well as of difference, between this method of using the grating and that customarily described in physics texts should be thoroughly understood by the student. In the latter method monochromatic light waves strike the grating and are deviated to right and left and then focused by a lens, forming real images of the source. In the present case the waves are deviated in exactly the same way but immediately afterwards enter the observer's eye. Accordingly, the eye sees the virtual images of the source on each side of the center. On the mathematical side the two methods are practically identical, but the one used here has the advantage of not requiring a lens.

In the actual apparatus the grating, scale, and light are all mounted on an optical bench and have sufficient freedom of motion to allow the necessary adjustments to be made. In order to secure accurate results, the instrument must be carefully set up and this is most conveniently done in the following order:

1. See that the scale is horizontal by comparing it with some horizontal line in the room, such as the top of the window casing. Place it perpendicular to the optical bench by making its ends equidistant from the end of the bench. This distance can be most easily measured and compared by using the wires fastened to the bench.

2. Place the grating so that it is approximately parallel to the scale, and, by looking through it, adjust the light so that the triangular slit is at the proper height to form the images on the scale and that the vertical line passing through the middle of the slit corresponds to the middle division of the scale.

3. Adjust the grating by rotating it about a horizontal axis parallel to the bench until the images are parallel to the scale.

4. Adjust the grating by rotating about a vertical axis until the images are equally spaced from the center.

5. When working with first-order images move the light source until all parallax between these images and the scale is eliminated. When working in second order, do the same with second-order images.

6. Determine the positions of the two first-order images and measure the length $AC$.

7. Shift the light and scale, so that data similar to that taken may be obtained for *two other* values of $AC$, thus making *three*

separate determinations. For one of these determinations use second-order images. If the adjustments have been carefully made for one position, they need not be repeated for the others, except the elimination of parallax. In making the readings be sure that the eye is held so that the slit is at the middle of the scale. Calculate the wave length of sodium light for each of the three pairs of measurements.

8. At one of the positions, close the opening in the Bunsen burner. Describe the resulting phenomena and explain. Note especially the order of the colors.

### Questions

**1.** Explain the action of a diffraction grating. Derive the equation used in this experiment, applying it also to second-order images.

**2.** Sketch Fig. 55 in the notebook, indicating the position of the eye and the actual path of a ray from the source to the eye.

**3.** Does the phenomenon of diffraction strengthen or weaken the argument for the wave theory of light? Explain.

**4.** Which color of the spectrum is deviated the most? How does this compare with the deviation due to a prism?

**5.** Take the grating to the Museum and explain the appearance of mercury light when viewed through it.

## EXPERIMENT 76

## Polarization of Light

Object: *To study some simple polarization phenomena.*

Since light waves are transverse in character, it is possible to polarize these waves, so that the vibrations take place in one plane. A simple form of polariscope is furnished with which some of the properties of such plane-polarized light will be studied. The hollow base of the polariscope (cover may be pried off with the fingers—turn little nick towards ground glass holder when replacing) contains the following articles: (1) pile of glass plates, (2) bit of mirror or polished brass, (3) plate of quartz in brass mounting, (4) mica plate, (5) glass plate in clamp. Please check these over when the instrument is received and report any deficiency to the instructor. Also check them carefully before returning the instrument to the desk.

1. The knurled head rotating over the divided circle at the upper end of the rod has mounted in it a Nicol prism. This is a polarizing agent which transmits only light waves vibrating (electric vector) *in the direction of the shortest diagonal.* To be transmitted by such a prism, the light vibration must, accordingly, be in the plane of this diagonal. If light vibrating in some other plane enters the Nicol, only the component of the vibration in this plane will be transmitted. Pull the knurled head out from the divided circle and the end of the Nicol can readily be seen.

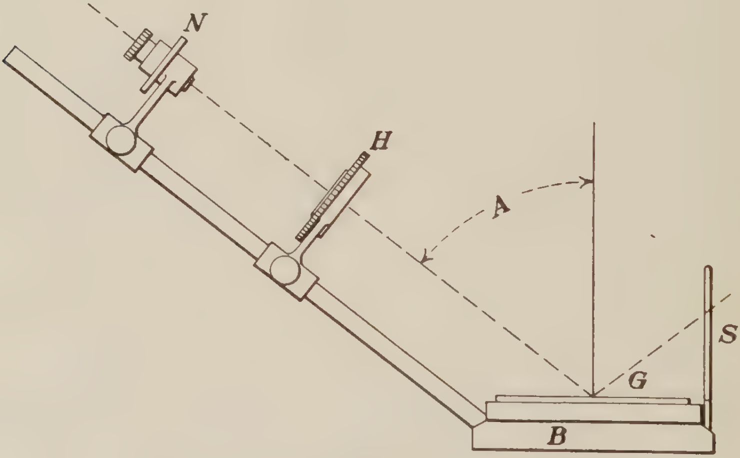


Fig. 56.

2. It is a property of glass and similar materials that light reflected from it at an oblique angle (for glass about 57 deg.) is plane-polarized. Work with the polariscope in a good light pointed towards a window—the specimen holder H may be turned out of the line of vision—and study the polarized light from the black glass plate.

The "plane of polarization" is a term frequently used in this connection. It is the plane at right angles to the plane of vibration.

3. Try other materials as polarizers—a mirror or bit of polished brass, and a pile of several pieces of clear glass.

4. Remove the Nicol holder from the divided circle and, holding it in the hand, study the polarization of light reflected from the black glass at more oblique and less oblique angles.

5. Holding the pile of plates in one hand and the Nicol in the other, look *through* this pile of plates at an oblique incidence (about 57 deg.) at some light object.

6. If the day is fair, look at the blue sky through the Nicol prism in a direction nearly at right angles to the sun.

7. Turn the specimen holder into position and set Nicol for extinction. Then insert the plate of quartz which is cut perpendicular to the axis. Use first a sodium burner as a source of light and then daylight. Describe and explain the phenomena. This piece of quartz is 4 mm. thick and gives for sodium light a rotation of very nearly 90 deg. With this light alone it is, accordingly, impossible to say whether the rotation is right-handed or left-handed.

8. Insert the mica plate. Describe and explain the phenomena, using sodium and also white light.

9. Insert glass in its clamp. Describe and explain phenomena when glass is unstrained and when strained.

### Questions

(The numbers of the following questions refer to the numbered paragraphs just preceding.)

**1.** Can polarized light be recognized with the unaided eye?

**2.** What is the plane of *vibration* of the reflected light? What is the plane of *polarization* of the reflected light?

**3.** Do all these materials polarize? What can be said about the character of the light reflected from the mirror or metal plate? Is the pile of plates superior to the black glass and, if so, in what way?

**4.** Is complete polarization obtained at any other angle?

**5.** What is the state of polarization of the light transmitted obliquely by the glass plates? If polarized, what is the plane of vibration?

**6.** Does the sky show any polarization, and, if so, what is the plane of vibration?

### *Optional*

**7.** Given the information that such a piece of quartz rotates blue light more than red, can a method be devised, using white light, which will enable one to say in which direction the rotation takes place?

**8.** No question.

**9.** How would strain be detected in a poorly annealed piece of glass? In a transparent (xylonite) model bridge girder?

# PART VII

# TABLES

## LOGARITHMS

**Table I. Logarithms.**—The common logarithm of a number is the power to which 10 must be raised to give this number. The advantage of logarithmic computation is that it substitutes for the arithmetical operations of multiplication and division the relatively simpler ones of addition and subtraction.

The use of the table will be explained by the following examples. If studied carefully, they should serve to make this system of computation available even for those who have no previous acquaintance with it.

*Example* 1.—Find the product 22 × 36. In the first column to the right of 22 the number .3424 is found, the decimal point being understood to belong before these figures. This is called the *mantissa* or fractional portion of the logarithm. Now, as 22 lies between 10 and 100 it is larger than 10 raised to the first power and smaller than 10 to the second. Accordingly, the number 1 is added as a *characteristic* and 1.3424 written as the logarithm of 22. Similarly, log 36 = 1.5563. Adding these two 2.8987 results as a logarithm of the product.

Now, looking in the table for 8987, it will be found that it is opposite 79 and under the number 2, *i.e.*, the *antilogarithm* or number corresponding to 2.8987 is 792, the correct value.

*Example* 2.—Find the product 13.7 × 4678. The logarithm of 13.7 is at once written down as 1.1367. That of 4678 cannot be found quite so simply. The mantissa of 4670 is .6693 and of 4680, .6702. By interpolation, that of 4678 is .6700. The nine columns at the right hand of the logarithm table will save time in this interpolation. Looking in the column headed by the fourth figure of the number, that is, 8 in the present case, it is seen that the number 7 is to be added to .6693 to get the logarithm wanted. As the number is greater than 1000 (*i.e.*, $10^3$) but less than 10000, the characteristic will be 3 and the whole logarithm is 3.6700.

The problem may be set down as follows:

$$\begin{aligned} \log 13.7 &= 1.1367 \\ \log 4678 &= 3.6700 \\ \hline \log \text{product } (X) &= 4.8067 \\ X &= 64070 \end{aligned}$$

This is as near the true result of 64088.6 as can be attained with this simple log table. The accuracy is ample for the experiments in this manual.

*Example* 3.—Find the value of $X$, if $X = 412396 \times .0324$.

$$\begin{aligned} \log 412400 &= 5.6153 \\ \log 0.0324 &= 8.5105 - 10 \\ \hline \log X &= 4.1258 \end{aligned}$$

$$X = 13360.$$

Since only four significant figures can be handled in this table, the first logarithm is that of 412400. The mantissa of the second logarithm, is, of course, that of 324, but to find the characteristic requires a little reasoning. Thus,

$$\begin{aligned} \log 324 &= 2.5105 \\ \log 32.4 &= 1.5105 \\ \log 3.24 &= 0.5105 \\ \log 0.324 &= \bar{1}.5105 \\ \log 0.0324 &= \bar{2}.5105 \\ &= 8.5105 - 10 \end{aligned}$$

The negative sign when used as above, *i.e.*, $\bar{1}.5105$, applies *only* to the characteristic. The mantissa is *always* positive.

*Example* 4.—Find $X$, if $X = 0.06373 \div 43.674$.

$$\begin{aligned} \log 0.06373 &= 8.8043 - 10 \\ \log 43.67 &= 1.6402 \\ \hline \text{By subtraction, } \log X &= 7.1641 - 10 \\ X &= 0.001459 \end{aligned}$$

*Example* 5.—Find $X$, if $X = 416.2^2$.

$$\begin{aligned} \log\ 416.2 &= 2.6193 \\ \text{Multiplied by } 2 &= 5.2386 \\ X &= 173200 \end{aligned}$$

*Example* 6.—Find $X$, if $X = \sqrt[3]{327.84}$.

$$\begin{aligned} \log\ 327.8 &= 2.5156 \\ \text{Divided by } 3 &= 0.8385 \\ X &= 6.895 \end{aligned}$$

*Example* 7.—Find $X$, if $X = \sqrt[3]{.0485}$.

$$\begin{aligned} \log 0.0485 &= 8.6857 - 10 \\ \text{Divided by } 3 &= 2.8952 - 3.3333 \\ &= 9.5619 - 10 \\ X &= 0.3647 \end{aligned}$$

The principles illustrated by the foregoing examples may be summarized as follows:

To multiply, add logarithms.

To divide, subtract logarithms.

To raise to the $n$th power, multiply logarithm by $n$.

To extract the $n$th root, divide logarithm by $n$.

If number is *greater* than 1, the characteristic of the logarithm is *one unit less* than the number of figures to the left of the decimal point.

If the number is *less than* 1, the characteristic of the logarithm is *negative* and *one unit more* than the *number of zeros* between the decimal point and the first significant figure of the given number.

**Table II. Reciprocals.**—If the reciprocal of 4.68 is desired, locate the line which begins with 4.6, and the number in that line under the column headed 8 gives the required number, which in this case is 0.2137. The position of the decimal point is indicated by the figure in the second column of each line. The reciprocal of 46.8 is 0.02137; hence, to find reciprocals of numbers having the same succession of figures as indicated in the table, but with the decimal point in a different position, shift the decimal point of the reciprocal as many places as it is shifted in the number but in the opposite direction.

Reciprocal 3.23 = 0.3096
Reciprocal 0.323 = 3.096
Reciprocal 323. = 0.003096
Reciprocal 0.000323 = 3096

**Table III. Squares.**—The following example illustrates how to use this table: The square of 316 is to be found on the line which begins with 31 and is in the column headed by 6; it is equal to 99856. To find the square of 31.6, the succession of figures is the same as for the square of 316, but there must be two decimal places in the result, which makes it equal to 998.56. In general, there must be twice as many decimal places in the square as in the number whose square is desired. For example:

$$
\begin{aligned}
258.^2 &= 66564\\
2.58^2 &= 6.6564\\
0.258^2 &= 0.066564
\end{aligned}
$$

**Table IV. Natural Sines.**—To find the sine of 16° 43′, locate the horizontal line which begins with 16° and then in the column headed by 42′ the number 2874 is found. When a decimal point is placed in front of it, it becomes the sine of 16° 42′. To find the sine 16° 43′, locate the figure in the same line, which, however is in the column headed by 1 and add this figure to the value previously found.

Thus 2874 + 3 = 2877
Hence the sine 16° 43′ = 0.2877.

In the same way, since 43° 17′ is found by adding 11 to 6845 or sine 43° 17′ = 0.6856.

**Table V. Natural Cosines.**—These are found by a method similar to that used in obtaining the sines, except that the number in the last two columns must be subtracted, not added. A decimal point must be placed in front of each of the numbers.

**Table VI. Natural Tangents.**—These are found in precisely the same way as the sines. The position of the decimal point for each line is indicated in the second column.

TABLE I.—LOGARITHMS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 0000 | 0043 | 0086 | 0128 | 0170 | 0212 | 0253 | 0294 | 0334 | 0374 | 4 | 8 | 12 | 17 | 21 | 25 | 29 | 33 | 37 |
| 11 | 0414 | 0453 | 0492 | 0531 | 0569 | 0607 | 0645 | 0682 | 0719 | 0755 | 4 | 8 | 11 | 15 | 19 | 23 | 26 | 30 | 34 |
| 12 | 0792 | 0828 | 0864 | 0899 | 0934 | 0969 | 1004 | 1038 | 1072 | 1106 | 3 | 7 | 10 | 14 | 17 | 21 | 24 | 28 | 31 |
| 13 | 1139 | 1173 | 1206 | 1239 | 1271 | 1303 | 1335 | 1367 | 1399 | 1430 | 3 | 6 | 10 | 13 | 16 | 19 | 23 | 26 | 29 |
| 14 | 1461 | 1492 | 1523 | 1553 | 1584 | 1614 | 1644 | 1673 | 1703 | 1732 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 |
| 15 | 1761 | 1790 | 1818 | 1847 | 1875 | 1903 | 1931 | 1959 | 1987 | 2014 | 3 | 6 | 8 | 11 | 14 | 17 | 20 | 22 | 25 |
| 16 | 2041 | 2068 | 2095 | 2122 | 2148 | 2175 | 2201 | 2227 | 2253 | 2279 | 3 | 5 | 8 | 11 | 13 | 16 | 18 | 21 | 24 |
| 17 | 2304 | 2330 | 2355 | 2380 | 2405 | 2430 | 2455 | 2480 | 2504 | 2529 | 2 | 5 | 7 | 10 | 12 | 15 | 17 | 20 | 22 |
| 18 | 2553 | 2577 | 2601 | 2625 | 2648 | 2672 | 2695 | 2718 | 2742 | 2765 | 2 | 5 | 7 | 9 | 12 | 14 | 16 | 19 | 21 |
| 19 | 2788 | 2810 | 2833 | 2856 | 2878 | 2900 | 2923 | 2945 | 2967 | 2989 | 2 | 4 | 7 | 9 | 11 | 13 | 16 | 18 | 20 |
| 20 | 3010 | 3032 | 3054 | 3075 | 3096 | 3118 | 3139 | 3160 | 3181 | 3201 | 2 | 4 | 6 | 8 | 11 | 13 | 15 | 17 | 19 |
| 21 | 3222 | 3243 | 3263 | 3284 | 3304 | 3324 | 3345 | 3365 | 3385 | 3404 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| 22 | 3424 | 3444 | 3464 | 3483 | 3502 | 3522 | 3541 | 3560 | 3579 | 3598 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 17 |
| 23 | 3617 | 3636 | 3655 | 3674 | 3692 | 3711 | 3729 | 3747 | 3766 | 3784 | 2 | 4 | 6 | 7 | 9 | 11 | 13 | 15 | 17 |
| 24 | 3802 | 3820 | 3838 | 3856 | 3874 | 3892 | 3909 | 3927 | 3945 | 3962 | 2 | 4 | 5 | 7 | 9 | 11 | 12 | 14 | 16 |
| 25 | 3979 | 3997 | 4014 | 4031 | 4048 | 4065 | 4082 | 4099 | 4116 | 4133 | 2 | 3 | 5 | 7 | 9 | 10 | 12 | 14 | 15 |
| 26 | 4150 | 4166 | 4183 | 4200 | 4216 | 4232 | 4249 | 4265 | 4281 | 4298 | 2 | 3 | 5 | 7 | 8 | 10 | 11 | 13 | 15 |
| 27 | 4314 | 4330 | 4346 | 4362 | 4378 | 4393 | 4409 | 4425 | 4440 | 4456 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 13 | 14 |
| 28 | 4472 | 4487 | 4502 | 4518 | 4533 | 4548 | 4564 | 4579 | 4594 | 4609 | 2 | 3 | 5 | 6 | 8 | 9 | 11 | 12 | 14 |
| 29 | 4624 | 4639 | 4654 | 4669 | 4683 | 4698 | 4713 | 4728 | 4742 | 4757 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 12 | 13 |
| 30 | 4771 | 4786 | 4800 | 4814 | 4829 | 4843 | 4857 | 4871 | 4886 | 4900 | 1 | 3 | 4 | 6 | 7 | 9 | 10 | 11 | 13 |
| 31 | 4914 | 4928 | 4942 | 4955 | 4969 | 4983 | 4997 | 5011 | 5024 | 5038 | 1 | 3 | 4 | 6 | 7 | 8 | 10 | 11 | 12 |
| 32 | 5051 | 5065 | 5079 | 5092 | 5105 | 5119 | 5132 | 5145 | 5159 | 5172 | 1 | 3 | 4 | 5 | 7 | 8 | 9 | 11 | 12 |
| 33 | 5185 | 5198 | 5211 | 5224 | 5237 | 5250 | 5263 | 5276 | 5289 | 5302 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 12 |
| 34 | 5315 | 5328 | 5340 | 5353 | 5366 | 5378 | 5391 | 5403 | 5416 | 5428 | 1 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 |
| 35 | 5441 | 5453 | 5465 | 5478 | 5490 | 5502 | 5514 | 5527 | 5539 | 5551 | 1 | 2 | 4 | 5 | 6 | 7 | 9 | 10 | 11 |
| 36 | 5563 | 5575 | 5587 | 5599 | 5611 | 5623 | 5635 | 5647 | 5658 | 5670 | 1 | 2 | 4 | 5 | 6 | 7 | 8 | 10 | 11 |
| 37 | 5682 | 5694 | 5705 | 5717 | 5729 | 5740 | 5752 | 5763 | 5775 | 5786 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 38 | 5798 | 5809 | 5821 | 5832 | 5843 | 5855 | 5866 | 5877 | 5888 | 5899 | 1 | 2 | 3 | 5 | 6 | 7 | 8 | 9 | 10 |
| 39 | 5911 | 5922 | 5933 | 5944 | 5955 | 5966 | 5977 | 5988 | 5999 | 6010 | 1 | 2 | 3 | 4 | 5 | 7 | 8 | 9 | 10 |
| 40 | 6021 | 6031 | 6042 | 6053 | 6064 | 6075 | 6085 | 6096 | 6107 | 6117 | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 |
| 41 | 6128 | 6138 | 6149 | 6160 | 6170 | 6180 | 6191 | 6201 | 6212 | 6222 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 42 | 6232 | 6243 | 6253 | 6263 | 6274 | 6284 | 6294 | 6304 | 6314 | 6325 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 43 | 6335 | 6345 | 6355 | 6365 | 6375 | 6385 | 6395 | 6405 | 6415 | 6425 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 44 | 6435 | 6444 | 6454 | 6464 | 6474 | 6484 | 6493 | 6503 | 6513 | 6522 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 45 | 6532 | 6542 | 6551 | 6561 | 6571 | 6580 | 6590 | 6599 | 6609 | 6618 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 46 | 6628 | 6637 | 6646 | 6656 | 6665 | 6675 | 6684 | 6693 | 6702 | 6712 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 7 | 8 |
| 47 | 6721 | 6730 | 6739 | 6749 | 6758 | 6767 | 6776 | 6785 | 6794 | 6803 | 1 | 2 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| 48 | 6812 | 6821 | 6830 | 6839 | 6848 | 6857 | 6866 | 6875 | 6884 | 6893 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 49 | 6902 | 6911 | 6920 | 6928 | 6937 | 6946 | 6955 | 6964 | 6972 | 6981 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 |
| 50 | 6990 | 6998 | 7007 | 7016 | 7024 | 7033 | 7042 | 7050 | 7059 | 7067 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 51 | 7076 | 7084 | 7093 | 7101 | 7110 | 7118 | 7126 | 7135 | 7143 | 7152 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 |
| 52 | 7160 | 7168 | 7177 | 7185 | 7193 | 7202 | 7210 | 7218 | 7226 | 7235 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 7 | 7 |
| 53 | 7243 | 7251 | 7259 | 7267 | 7275 | 7284 | 7292 | 7300 | 7308 | 7316 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |
| 54 | 7324 | 7332 | 7340 | 7348 | 7356 | 7364 | 7372 | 7380 | 7388 | 7396 | 1 | 2 | 2 | 3 | 4 | 5 | 6 | 6 | 7 |

Table I.—Logarithms—*Continued*

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 7404 | 7412 | 7419 | 7427 | 7435 | 7443 | 7451 | 7459 | 7466 | 7474 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 56 | 7482 | 7490 | 7497 | 7505 | 7513 | 7520 | 7528 | 7536 | 7543 | 7551 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 57 | 7559 | 7566 | 7574 | 7582 | 7589 | 7597 | 7604 | 7612 | 7619 | 7627 | 1 | 2 | 2 | 3 | 4 | 5 | 5 | 6 | 7 |
| 58 | 7634 | 7642 | 7649 | 7657 | 7664 | 7672 | 7679 | 7686 | 7694 | 7701 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 59 | 7709 | 7716 | 7723 | 7731 | 7738 | 7745 | 7752 | 7760 | 7767 | 7774 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 |
| 60 | 7782 | 7789 | 7796 | 7803 | 7810 | 7818 | 7825 | 7832 | 7839 | 7846 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 61 | 7853 | 7860 | 7868 | 7875 | 7882 | 7889 | 7896 | 7903 | 7910 | 7917 | 1 | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 6 |
| 62 | 7924 | 7931 | 7938 | 7945 | 7952 | 7959 | 7966 | 7973 | 7980 | 7987 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 6 | 6 |
| 63 | 7993 | 8000 | 8007 | 8014 | 8021 | 8028 | 8035 | 8041 | 8048 | 8055 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 64 | 8062 | 8069 | 8075 | 8082 | 8089 | 8096 | 8102 | 8109 | 8116 | 8122 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 65 | 8129 | 8136 | 8142 | 8149 | 8156 | 8162 | 8169 | 8176 | 8182 | 8189 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 66 | 8195 | 8202 | 8209 | 8215 | 8222 | 8228 | 8235 | 8241 | 8248 | 8254 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 67 | 8261 | 8267 | 8274 | 8280 | 8287 | 8293 | 8299 | 8306 | 8312 | 8319 | 1 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 |
| 68 | 8325 | 8331 | 8338 | 8344 | 8351 | 8357 | 8363 | 8370 | 8376 | 8382 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 6 |
| 69 | 8388 | 8395 | 8401 | 8407 | 8414 | 8420 | 8426 | 8432 | 8439 | 8445 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 70 | 8451 | 8457 | 8463 | 8470 | 8476 | 8482 | 8488 | 8494 | 8500 | 8506 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 |
| 71 | 8513 | 8519 | 8525 | 8531 | 8537 | 8543 | 8549 | 8555 | 8561 | 8567 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 72 | 8573 | 8579 | 8585 | 8591 | 8597 | 8603 | 8609 | 8615 | 8621 | 8627 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 73 | 8633 | 8639 | 8645 | 8651 | 8657 | 8663 | 8669 | 8675 | 8681 | 8686 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 74 | 8692 | 8698 | 8704 | 8710 | 8716 | 8722 | 8727 | 8733 | 8739 | 8745 | 1 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 |
| 75 | 8751 | 8756 | 8762 | 8768 | 8774 | 8779 | 8785 | 8791 | 8797 | 8802 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 76 | 8808 | 8814 | 8820 | 8825 | 8831 | 8837 | 8842 | 8848 | 8854 | 8859 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 5 |
| 77 | 8865 | 8871 | 8876 | 8882 | 8887 | 8893 | 8899 | 8904 | 8910 | 8915 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 78 | 8921 | 8927 | 8932 | 8938 | 8943 | 8949 | 8954 | 8960 | 8965 | 8971 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 79 | 8976 | 8982 | 8987 | 8993 | 8998 | 9004 | 9009 | 9015 | 9020 | 9025 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 80 | 9031 | 9036 | 9042 | 9047 | 9053 | 9058 | 9063 | 9069 | 9074 | 9079 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 81 | 9085 | 9090 | 9096 | 9101 | 9106 | 9112 | 9117 | 9122 | 9128 | 9133 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 82 | 9138 | 9143 | 9149 | 9154 | 9159 | 9165 | 9170 | 9175 | 9180 | 9186 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 83 | 9191 | 9196 | 9201 | 9206 | 9212 | 9217 | 9222 | 9227 | 9232 | 9238 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 84 | 9243 | 9248 | 9253 | 9258 | 9263 | 9269 | 9274 | 9279 | 9284 | 9289 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 85 | 9294 | 9299 | 9304 | 9309 | 9315 | 9320 | 9325 | 9330 | 9335 | 9340 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 86 | 9345 | 9350 | 9355 | 9360 | 9365 | 9370 | 9375 | 9380 | 9385 | 9390 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 |
| 87 | 9395 | 9400 | 9405 | 9410 | 9415 | 9420 | 9425 | 9430 | 9435 | 9440 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 88 | 9445 | 9450 | 9455 | 9460 | 9465 | 9469 | 9474 | 9479 | 9484 | 9489 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 89 | 9494 | 9499 | 9504 | 9509 | 9513 | 9518 | 9523 | 9528 | 9533 | 9538 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 90 | 9542 | 9547 | 9552 | 9557 | 9562 | 9566 | 9571 | 9576 | 9581 | 9586 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 91 | 9590 | 9595 | 9600 | 9605 | 9609 | 9614 | 9619 | 9624 | 9628 | 9633 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 92 | 9638 | 9643 | 9647 | 9652 | 9657 | 9661 | 9666 | 9671 | 9675 | 9680 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 93 | 9685 | 9689 | 9694 | 9699 | 9703 | 9708 | 9713 | 9717 | 9722 | 9727 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 94 | 9731 | 9736 | 9741 | 9745 | 9750 | 9754 | 9759 | 9763 | 9768 | 9773 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 95 | 9777 | 9782 | 9786 | 9791 | 9795 | 9800 | 9805 | 9809 | 9814 | 9818 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 96 | 9823 | 9827 | 9832 | 9836 | 9841 | 9845 | 9850 | 9854 | 9859 | 9863 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 97 | 9868 | 9872 | 9877 | 9881 | 9886 | 9890 | 9894 | 9899 | 9903 | 9908 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 98 | 9912 | 9917 | 9921 | 9926 | 9930 | 9934 | 9939 | 9943 | 9948 | 9952 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 4 |
| 99 | 9956 | 9961 | 9965 | 9969 | 9974 | 9978 | 9983 | 9987 | 9991 | 9996 | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 3 | 4 |

### TABLE II.—RECIPROCALS

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | ....... | 100.0 | 50.00 | 33.33 | 25.00 | 20.00 | 16.67 | 14.29 | 12.50 | 11.11 |
| .1 | 10.0000 | 9.091 | 8.333 | 7.692 | 7.143 | 6.667 | 6.250 | 5.882 | 5.556 | 5.263 |
| .2 | 5.0000 | 4.762 | 4.545 | 4.348 | 4.167 | 4.000 | 3.846 | 3.704 | 3.571 | 3.448 |
| .3 | 3.3333 | 3.226 | 3.125 | 3.030 | 2.941 | 2.857 | 2.778 | 2.703 | 2.632 | 2.564 |
| .4 | 2.5000 | 2.439 | 2.381 | 2.326 | 2.273 | 2.222 | 2.174 | 2.128 | 2.083 | 2.041 |
| .5 | 2.0000 | *9608 | *9231 | *8868 | *8519 | *8182 | *7857 | *7544 | *7241 | *6949 |
| .6 | 1.6667 | 6393 | 6129 | 5873 | 5625 | 5385 | 5152 | 4925 | 4706 | 4493 |
| .7 | 4286 | 4085 | 3889 | 3699 | 3514 | 3333 | 3158 | 2987 | 2821 | 2658 |
| .8 | 2500 | 2346 | 2195 | 2048 | 1905 | 1765 | 1628 | 1494 | 1364 | 1236 |
| .9 | 1111 | 0989 | 0870 | 0753 | 0638 | 0526 | 0417 | 0309 | 0204 | 0101 |
| 1.0 | 1.0000 | *9901 | *9804 | *9709 | *9615 | *9524 | *9434 | *9346 | *9259 | *9174 |
| 1.1 | 0.9091 | 9009 | 8929 | 8850 | 8772 | 8696 | 8621 | 8547 | 8475 | 8403 |
| 1.2 | 8333 | 8264 | 8197 | 8130 | 8065 | 8000 | 7937 | 7874 | 7813 | 7752 |
| 1.3 | 7692 | 7634 | 7576 | 7519 | 7463 | 7407 | 7353 | 7299 | 7246 | 7194 |
| 1.4 | 7143 | 7092 | 7042 | 6993 | 6944 | 6897 | 6849 | 6803 | 6757 | 6711 |
| 1.5 | 6667 | 6623 | 6579 | 6536 | 6494 | 6452 | 6410 | 6369 | 6329 | 6289 |
| 1.6 | 6250 | 6211 | 6173 | 6135 | 6098 | 6061 | 6024 | 5988 | 5952 | 5917 |
| 1.7 | 5882 | 5848 | 5814 | 5780 | 5747 | 5714 | 5682 | 5650 | 5618 | 5587 |
| 1.8 | 5556 | 5525 | 5495 | 5464 | 5435 | 5405 | 5376 | 5348 | 5319 | 5291 |
| 1.9 | 5263 | 5236 | 5208 | 5181 | 5155 | 5128 | 5102 | 5076 | 5051 | 5025 |
| 2.0 | 0.5000 | 4975 | 4950 | 4926 | 4902 | 4878 | 4854 | 4831 | 4808 | 4785 |
| 2.1 | 4762 | 4739 | 4717 | 4695 | 4673 | 4651 | 4630 | 4608 | 4587 | 4566 |
| 2.2 | 4545 | 4525 | 4505 | 4484 | 4464 | 4444 | 4425 | 4405 | 4386 | 4367 |
| 2.3 | 4348 | 4329 | 4310 | 4292 | 4274 | 4255 | 4237 | 4219 | 4202 | 4184 |
| 2.4 | 4167 | 4149 | 4132 | 4115 | 4098 | 4082 | 4065 | 4049 | 4032 | 4016 |
| 2.5 | 4000 | 3984 | 3968 | 3953 | 3937 | 3922 | 3906 | 3891 | 3876 | 3861 |
| 2.6 | 3846 | 3831 | 3817 | 3802 | 3788 | 3774 | 3759 | 3745 | 3731 | 3717 |
| 2.7 | 3704 | 3690 | 3676 | 3663 | 3650 | 3636 | 3623 | 3610 | 3597 | 3584 |
| 2.8 | 3571 | 3559 | 3546 | 3534 | 3521 | 3509 | 3497 | 3484 | 3472 | 3460 |
| 2.9 | 3448 | 3436 | 3425 | 3413 | 3401 | 3390 | 3378 | 3367 | 3356 | 3344 |
| 3.0 | 0.3333 | 3322 | 3311 | 3300 | 3289 | 3279 | 3268 | 3257 | 3247 | 3236 |
| 3.1 | 3226 | 3215 | 3205 | 3195 | 3185 | 3175 | 3165 | 3155 | 3145 | 3135 |
| 3.2 | 3125 | 3115 | 3106 | 3096 | 3086 | 3077 | 3067 | 3058 | 3049 | 3040 |
| 3.3 | 3030 | 3021 | 3012 | 3003 | 2994 | 2985 | 2976 | 2967 | 2959 | 2950 |
| 3.4 | 2941 | 2933 | 2924 | 2915 | 2907 | 2899 | 2890 | 2882 | 2874 | 2865 |
| 3.5 | 2857 | 2849 | 2841 | 2833 | 2825 | 2817 | 2809 | 2801 | 2793 | 2786 |
| 3.6 | 2778 | 2770 | 2762 | 2755 | 2747 | 2740 | 2732 | 2725 | 2717 | 2710 |
| 3.7 | 2703 | 2695 | 2688 | 2681 | 2674 | 2667 | 2660 | 2653 | 2646 | 2639 |
| 3.8 | 2632 | 2625 | 2618 | 2611 | 2604 | 2597 | 2591 | 2584 | 2577 | 2571 |
| 3.9 | 2564 | 2558 | 2551 | 2545 | 2538 | 2532 | 2525 | 2519 | 2513 | 2506 |
| 4.0 | 0.2500 | 2494 | 2488 | 2481 | 2475 | 2469 | 2463 | 2457 | 2451 | 2445 |
| 4.1 | 2439 | 2433 | 2427 | 2421 | 2415 | 2410 | 2404 | 2398 | 2392 | 2387 |
| 4.2 | 2381 | 2375 | 2370 | 2364 | 2358 | 2353 | 2347 | 2342 | 2336 | 2331 |
| 4.3 | 2326 | 2320 | 2315 | 2309 | 2304 | 2299 | 2294 | 2288 | 2283 | 2278 |
| 4.4 | 2273 | 2268 | 2262 | 2257 | 2252 | 2247 | 2242 | 2237 | 2232 | 2227 |
| 4.5 | 2222 | 2217 | 2212 | 2208 | 2203 | 2198 | 2193 | 2188 | 2183 | 2179 |
| 4.6 | 2174 | 2169 | 2165 | 2160 | 2155 | 2151 | 2146 | 2141 | 2137 | 2132 |
| 4.7 | 2128 | 2123 | 2119 | 2114 | 2110 | 2105 | 2101 | 2096 | 2092 | 2088 |
| 4.8 | 2083 | 2079 | 2075 | 2070 | 2066 | 2062 | 2058 | 2053 | 2049 | 2045 |
| 4.9 | 2041 | 2037 | 2033 | 2028 | 2024 | 2020 | 2016 | 2012 | 2008 | 2004 |

| | Differences | | | | |
|---|---|---|---|---|---|
| | 98 | 96 | 94 | 92 | 90 |
| 1 | 10 | 10 | 9 | 9 | 9 |
| 2 | 20 | 19 | 19 | 18 | 18 |
| 3 | 29 | 29 | 28 | 28 | 27 |
| 4 | 39 | 38 | 38 | 37 | 36 |
| 5 | 49 | 48 | 47 | 46 | 45 |
| 6 | 59 | 58 | 56 | 55 | 54 |
| 7 | 69 | 67 | 66 | 64 | 63 |
| 8 | 78 | 77 | 75 | 74 | 72 |
| 9 | 88 | 86 | 85 | 83 | 81 |
| | 88 | 86 | 84 | 82 | 80 |
| 1 | 9 | 9 | 8 | 8 | 8 |
| 2 | 18 | 17 | 17 | 16 | 16 |
| 3 | 26 | 26 | 25 | 25 | 24 |
| 4 | 35 | 34 | 34 | 33 | 32 |
| 5 | 44 | 43 | 42 | 41 | 40 |
| 6 | 53 | 52 | 50 | 49 | 48 |
| 7 | 62 | 60 | 59 | 57 | 56 |
| 8 | 70 | 69 | 67 | 66 | 64 |
| 9 | 79 | 77 | 76 | 74 | 72 |
| | 78 | 76 | 74 | 72 | 70 |
| 1 | 8 | 8 | 7 | 7 | 7 |
| 2 | 16 | 15 | 15 | 14 | 14 |
| 3 | 23 | 23 | 22 | 22 | 21 |
| 4 | 31 | 30 | 30 | 29 | 28 |
| 5 | 39 | 38 | 37 | 36 | 35 |
| 6 | 47 | 46 | 44 | 43 | 42 |
| 7 | 55 | 53 | 52 | 50 | 49 |
| 8 | 62 | 61 | 59 | 58 | 56 |
| 9 | 70 | 68 | 67 | 65 | 63 |
| | 68 | 66 | 64 | 62 | 60 |
| 1 | 7 | 7 | 6 | 6 | 6 |
| 2 | 14 | 13 | 13 | 12 | 12 |
| 3 | 20 | 20 | 19 | 19 | 18 |
| 4 | 27 | 26 | 26 | 25 | 24 |
| 5 | 34 | 33 | 32 | 31 | 30 |
| 6 | 41 | 40 | 38 | 37 | 36 |
| 7 | 48 | 46 | 45 | 43 | 42 |
| 8 | 54 | 53 | 51 | 50 | 48 |
| 9 | 61 | 59 | 58 | 56 | 54 |
| | 58 | 56 | 54 | 52 | 50 |
| 1 | 6 | 6 | 5 | 5 | 5 |
| 2 | 12 | 11 | 11 | 10 | 10 |
| 3 | 17 | 17 | 16 | 16 | 15 |
| 4 | 23 | 22 | 22 | 21 | 20 |
| 5 | 29 | 28 | 27 | 26 | 25 |
| 6 | 35 | 34 | 32 | 31 | 30 |
| 7 | 41 | 39 | 38 | 36 | 35 |
| 8 | 46 | 45 | 43 | 42 | 40 |
| 9 | 52 | 50 | 49 | 47 | 45 |
| | 48 | 46 | 44 | 43 | 42 |
| 1 | 5 | 5 | 4 | 4 | 4 |
| 2 | 10 | 9 | 9 | 9 | 8 |
| 3 | 14 | 14 | 13 | 13 | 13 |
| 4 | 19 | 18 | 18 | 17 | 17 |
| 5 | 24 | 23 | 22 | 22 | 21 |
| 6 | 29 | 28 | 26 | 26 | 25 |
| 7 | 34 | 32 | 31 | 30 | 29 |
| 8 | 38 | 37 | 35 | 34 | 34 |
| 9 | 43 | 41 | 40 | 39 | 38 |
| | 41 | 40 | 39 | 38 | 37 |
| 1 | 4 | 4 | 4 | 4 | 4 |
| 2 | 8 | 8 | 8 | 8 | 7 |
| 3 | 12 | 12 | 12 | 11 | 11 |
| 4 | 16 | 16 | 16 | 15 | 15 |
| 5 | 21 | 20 | 20 | 19 | 19 |
| 6 | 25 | 24 | 23 | 23 | 22 |
| 7 | 29 | 28 | 27 | 27 | 26 |
| 8 | 33 | 32 | 31 | 30 | 30 |
| 9 | 37 | 36 | 35 | 34 | 33 |

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* From Jones' "Logarithmic Tables."

### Table II.—Reciprocals—*Continued*

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 5.0 | 0.2000 | 1996 | 1992 | 1988 | 1984 | 1980 | 1976 | 1972 | 1969 | 1965 |
| 5.1 | 1961 | 1957 | 1953 | 1949 | 1946 | 1942 | 1938 | 1934 | 1931 | 1927 |
| 5.2 | 1923 | 1919 | 1916 | 1912 | 1908 | 1905 | 1901 | 1898 | 1894 | 1890 |
| 5.3 | 1887 | 1883 | 1880 | 1876 | 1873 | 1869 | 1866 | 1862 | 1859 | 1855 |
| 5.4 | 1852 | 1848 | 1845 | 1842 | 1838 | 1835 | 1832 | 1828 | 1825 | 1821 |
| 5.5 | 1818 | 1815 | 1812 | 1808 | 1805 | 1802 | 1799 | 1795 | 1792 | 1789 |
| 5.6 | 1786 | 1783 | 1779 | 1776 | 1773 | 1770 | 1767 | 1764 | 1761 | 1757 |
| 5.7 | 1754 | 1751 | 1748 | 1745 | 1742 | 1739 | 1736 | 1733 | 1730 | 1727 |
| 5.8 | 1724 | 1721 | 1718 | 1715 | 1712 | 1709 | 1706 | 1704 | 1701 | 1698 |
| 5.9 | 1695 | 1692 | 1689 | 1686 | 1684 | 1681 | 1678 | 1675 | 1672 | 1669 |
| 6.0 | 0.1667 | 1664 | 1661 | 1658 | 1656 | 1653 | 1650 | 1647 | 1645 | 1642 |
| 6.1 | 1639 | 1637 | 1634 | 1631 | 1629 | 1626 | 1623 | 1621 | 1618 | 1616 |
| 6.2 | 1613 | 1610 | 1608 | 1605 | 1603 | 1600 | 1597 | 1595 | 1592 | 1590 |
| 6.3 | 1587 | 1585 | 1582 | 1580 | 1577 | 1575 | 1572 | 1570 | 1567 | 1565 |
| 6.4 | 1563 | 1560 | 1558 | 1555 | 1553 | 1550 | 1548 | 1546 | 1543 | 1541 |
| 6.5 | 1538 | 1536 | 1534 | 1531 | 1529 | 1527 | 1524 | 1522 | 1520 | 1517 |
| 6.6 | 1515 | 1513 | 1511 | 1508 | 1506 | 1504 | 1502 | 1499 | 1497 | 1495 |
| 6.7 | 1493 | 1490 | 1488 | 1486 | 1484 | 1481 | 1479 | 1477 | 1475 | 1473 |
| 6.8 | 1471 | 1468 | 1466 | 1464 | 1462 | 1460 | 1458 | 1456 | 1453 | 1451 |
| 6.9 | 1449 | 1447 | 1445 | 1443 | 1441 | 1439 | 1437 | 1435 | 1433 | 1431 |
| 7.0 | 0.1429 | 1427 | 1425 | 1422 | 1420 | 1418 | 1416 | 1414 | 1412 | 1410 |
| 7.1 | 1408 | 1406 | 1404 | 1403 | 1401 | 1399 | 1397 | 1395 | 1393 | 1391 |
| 7.2 | 1389 | 1387 | 1385 | 1383 | 1381 | 1379 | 1377 | 1376 | 1374 | 1372 |
| 7.3 | 1370 | 1368 | 1366 | 1364 | 1362 | 1361 | 1359 | 1357 | 1355 | 1353 |
| 7.4 | 1351 | 1350 | 1348 | 1346 | 1344 | 1342 | 1340 | 1339 | 1337 | 1335 |
| 7.5 | 1333 | 1332 | 1330 | 1328 | 1326 | 1325 | 1323 | 1321 | 1319 | 1318 |
| 7.6 | 1316 | 1314 | 1312 | 1311 | 1309 | 1307 | 1305 | 1304 | 1302 | 1300 |
| 7.7 | 1299 | 1297 | 1295 | 1294 | 1292 | 1290 | 1289 | 1287 | 1285 | 1284 |
| 7.8 | 1282 | 1280 | 1279 | 1277 | 1276 | 1274 | 1272 | 1271 | 1269 | 1267 |
| 7.9 | 1266 | 1264 | 1263 | 1261 | 1259 | 1258 | 1256 | 1255 | 1253 | 1252 |
| 8.0 | 0.1250 | 1248 | 1247 | 1245 | 1244 | 1242 | 1241 | 1239 | 1238 | 1236 |
| 8.1 | 1235 | 1233 | 1232 | 1230 | 1229 | 1227 | 1225 | 1224 | 1222 | 1221 |
| 8.2 | 1220 | 1218 | 1217 | 1215 | 1214 | 1212 | 1211 | 1209 | 1208 | 1206 |
| 8.3 | 1205 | 1203 | 1202 | 1200 | 1199 | 1198 | 1196 | 1195 | 1193 | 1192 |
| 8.4 | 1190 | 1189 | 1188 | 1186 | 1185 | 1183 | 1182 | 1181 | 1179 | 1178 |
| 8.5 | 1176 | 1175 | 1174 | 1172 | 1171 | 1170 | 1168 | 1167 | 1166 | 1164 |
| 8.6 | 1163 | 1161 | 1160 | 1159 | 1157 | 1156 | 1155 | 1153 | 1152 | 1151 |
| 8.7 | 1149 | 1148 | 1147 | 1145 | 1144 | 1143 | 1142 | 1140 | 1139 | 1138 |
| 8.8 | 1136 | 1135 | 1134 | 1133 | 1131 | 1130 | 1129 | 1127 | 1126 | 1125 |
| 8.9 | 1124 | 1122 | 1121 | 1120 | 1119 | 1117 | 1116 | 1115 | 1114 | 1112 |
| 9.0 | 0.1111 | 1110 | 1109 | 1107 | 1106 | 1105 | 1104 | 1103 | 1101 | 1100 |
| 9.1 | 1099 | 1098 | 1096 | 1095 | 1094 | 1093 | 1092 | 1091 | 1089 | 1088 |
| 9.2 | 1087 | 1086 | 1085 | 1083 | 1082 | 1081 | 1080 | 1079 | 1078 | 1076 |
| 9.3 | 1075 | 1074 | 1073 | 1072 | 1071 | 1070 | 1068 | 1067 | 1066 | 1065 |
| 9.4 | 1064 | 1063 | 1062 | 1060 | 1059 | 1058 | 1057 | 1056 | 1055 | 1054 |
| 9.5 | 1053 | 1052 | 1050 | 1049 | 1048 | 1047 | 1046 | 1045 | 1044 | 1043 |
| 9.6 | 1042 | 1041 | 1040 | 1038 | 1037 | 1036 | 1035 | 1034 | 1033 | 1032 |
| 9.7 | 1031 | 1030 | 1029 | 1028 | 1027 | 1026 | 1025 | 1024 | 1022 | 1021 |
| 9.8 | 1020 | 1019 | 1018 | 1017 | 1016 | 1015 | 1014 | 1013 | 1012 | 1011 |
| 9.9 | 1010 | 1009 | 1008 | 1007 | 1006 | 1005 | 1004 | 1003 | 1002 | 1001 |

| Differences | | | | | |
|---|---|---|---|---|---|
| | 36 | 35 | 34 | 33 | 32 |
| 1 | 4 | 4 | 3 | 3 | 3 |
| 2 | 7 | 7 | 7 | 7 | 6 |
| 3 | 11 | 11 | 10 | 10 | 10 |
| 4 | 14 | 14 | 14 | 13 | 13 |
| 5 | 18 | 18 | 17 | 17 | 16 |
| 6 | 22 | 21 | 20 | 20 | 19 |
| 7 | 25 | 25 | 24 | 23 | 22 |
| 8 | 29 | 28 | 27 | 26 | 26 |
| 9 | 32 | 32 | 31 | 30 | 29 |
| | 31 | 30 | 29 | 28 | 27 |
| 1 | 3 | 3 | 3 | 3 | 3 |
| 2 | 6 | 6 | 6 | 6 | 5 |
| 3 | 9 | 9 | 9 | 8 | 8 |
| 4 | 12 | 12 | 12 | 11 | 11 |
| 5 | 16 | 15 | 15 | 14 | 14 |
| 6 | 19 | 18 | 17 | 17 | 16 |
| 7 | 22 | 21 | 20 | 20 | 19 |
| 8 | 25 | 24 | 23 | 22 | 22 |
| 9 | 28 | 27 | 26 | 25 | 24 |
| | 26 | 25 | 24 | 23 | 22 |
| 1 | 3 | 3 | 2 | 2 | 2 |
| 2 | 5 | 5 | 5 | 5 | 4 |
| 3 | 8 | 8 | 7 | 7 | 7 |
| 4 | 10 | 10 | 10 | 9 | 9 |
| 5 | 13 | 13 | 12 | 12 | 11 |
| 6 | 16 | 15 | 14 | 14 | 13 |
| 7 | 18 | 18 | 17 | 16 | 15 |
| 8 | 21 | 20 | 19 | 18 | 18 |
| 9 | 23 | 23 | 22 | 21 | 20 |
| | 21 | 20 | 19 | 18 | 17 |
| 1 | 2 | 2 | 2 | 2 | 2 |
| 2 | 4 | 4 | 4 | 4 | 3 |
| 3 | 6 | 6 | 6 | 5 | 5 |
| 4 | 8 | 8 | 8 | 7 | 7 |
| 5 | 11 | 10 | 10 | 9 | 9 |
| 6 | 13 | 12 | 11 | 11 | 10 |
| 7 | 15 | 14 | 13 | 13 | 12 |
| 8 | 17 | 16 | 15 | 14 | 14 |
| 9 | 19 | 18 | 17 | 16 | 15 |
| | 10 | 15 | 14 | 13 | 12 |
| 1 | 2 | 2 | 1 | 1 | 1 |
| 2 | 3 | 3 | 3 | 3 | 2 |
| 3 | 5 | 5 | 4 | 4 | 4 |
| 4 | 6 | 6 | 6 | 5 | 5 |
| 5 | 8 | 8 | 7 | 7 | 6 |
| 6 | 10 | 9 | 8 | 8 | 7 |
| 7 | 11 | 11 | 10 | 9 | 8 |
| 8 | 13 | 12 | 11 | 10 | 10 |
| 9 | 14 | 14 | 13 | 12 | 11 |
| | 11 | 10 | 9 | 8 | 7 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 2 | 2 | 2 | 1 |
| 3 | 3 | 3 | 3 | 2 | 2 |
| 4 | 4 | 4 | 4 | 3 | 3 |
| 5 | 6 | 5 | 5 | 4 | 4 |
| 6 | 7 | 6 | 5 | 5 | 4 |
| 7 | 8 | 7 | 6 | 6 | 5 |
| 8 | 9 | 8 | 7 | 6 | 6 |
| 9 | 10 | 9 | 8 | 7 | 6 |
| | 6 | 5 | 4 | 3 | 2 |
| 1 | 1 | 1 | 0 | 0 | 0 |
| 2 | 1 | 1 | 1 | 1 | 0 |
| 3 | 2 | 2 | 1 | 1 | 1 |
| 4 | 2 | 2 | 2 | 1 | 1 |
| 5 | 3 | 3 | 2 | 2 | 1 |
| 6 | 4 | 3 | 2 | 2 | 1 |
| 7 | 4 | 4 | 3 | 2 | 1 |
| 8 | 5 | 4 | 3 | 2 | 2 |
| 9 | 5 | 5 | 4 | 3 | 2 |

## TABLE III.—SQUARES*

| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |
| 1 | 100 | 121 | 144 | 169 | 196 | 225 | 256 | 289 | 324 | 361 |
| 2 | 400 | 441 | 484 | 529 | 576 | 625 | 676 | 729 | 784 | 841 |
| 3 | 900 | 961 | 1024 | 1089 | 1156 | 1225 | 1296 | 1369 | 1444 | 1521 |
| 4 | 1600 | 1681 | 1764 | 1849 | 1936 | 2025 | 2116 | 2209 | 2304 | 2401 |
| 5 | 2500 | 2601 | 2704 | 2809 | 2916 | 3025 | 3136 | 3249 | 3364 | 3481 |
| 6 | 3600 | 3721 | 3844 | 3969 | 4096 | 4225 | 4356 | 4489 | 4624 | 4761 |
| 7 | 4900 | 5041 | 5184 | 5329 | 5476 | 5625 | 5776 | 5929 | 6084 | 6241 |
| 8 | 6400 | 6561 | 6724 | 6889 | 7056 | 7225 | 7396 | 7569 | 7744 | 7921 |
| 9 | 8100 | 8281 | 8464 | 8649 | 8836 | 9025 | 9216 | 9409 | 9604 | 9801 |
| 10 | 1 0000 | 1 0201 | 1 0404 | 1 0609 | 1 0816 | 1 1025 | 1 1236 | 1 1449 | 1 1664 | 1 1881 |
| 11 | 2100 | 2321 | 2544 | 2769 | 2996 | 3225 | 3456 | 3689 | 3924 | 4161 |
| 12 | 4400 | 4641 | 4884 | 5129 | 5376 | 5625 | 5876 | 6129 | 6384 | 6641 |
| 13 | 6900 | 7161 | 7424 | 7689 | 7956 | 8225 | 8496 | 8769 | 9044 | 9321 |
| 14 | 9600 | 9881 | 2 0164 | 2 0449 | 2 0736 | 2 1025 | 2 1316 | 2 1609 | 2 1904 | 2 2201 |
| 15 | 2 2500 | 2 2801 | 3104 | 3409 | 3716 | 4025 | 4336 | 4649 | 4964 | 5281 |
| 16 | 5600 | 5921 | 6244 | 6569 | 6896 | 7225 | 7556 | 7889 | 8224 | 8561 |
| 17 | 8900 | 9241 | 9584 | 9929 | 3 0276 | 3 0625 | 3 0976 | 3 1329 | 3 1684 | 3 2041 |
| 18 | 3 2400 | 3 2761 | 3 3124 | 3 3489 | 3856 | 4225 | 4596 | 4969 | 5344 | 5721 |
| 19 | 6100 | 6481 | 6864 | 7249 | 7636 | 8025 | 8416 | 8809 | 9204 | 9601 |
| 20 | 4 0000 | 4 0401 | 4 0804 | 4 1209 | 4 1616 | 4 2025 | 4 2436 | 4 2849 | 4 3264 | 4 3681 |
| 21 | 4100 | 4521 | 4944 | 5369 | 5796 | 6225 | 6656 | 7089 | 7524 | 7961 |
| 22 | 8400 | 8841 | 9284 | 9729 | 5 0176 | 5 0625 | 5 1076 | 5 1529 | 5 1984 | 5 2441 |
| 23 | 5 2900 | 5 3361 | 5 3824 | 5 4289 | 4756 | 5225 | 5696 | 6169 | 6644 | 7121 |
| 24 | 7600 | 8081 | 8564 | 9049 | 9536 | 6 0025 | 6 0516 | 6 1009 | 6 1504 | 6 2001 |
| 25 | 6 2500 | 6 3001 | 6 3504 | 6 4009 | 6 4516 | 5025 | 5536 | 6049 | 6564 | 7081 |
| 26 | 7600 | 8121 | 8644 | 9169 | 9696 | 7 0225 | 7 0756 | 7 1289 | 7 1824 | 7 2361 |
| 27 | 7 2900 | 7 3441 | 7 3984 | 7 4529 | 7 5076 | 5625 | 6176 | 6729 | 7284 | 7841 |
| 28 | 8400 | 8961 | 9524 | 8 0089 | 8 0656 | 8 1225 | 8 1796 | 8 2369 | 8 2944 | 8 3521 |
| 29 | 8 4100 | 8 4681 | 8 5264 | 5849 | 6436 | 7025 | 7616 | 8209 | 8804 | 9401 |
| 30 | 9 0000 | 9 0601 | 9 1204 | 9 1809 | 9 2416 | 9 3025 | 9 3636 | 9 4249 | 9 4864 | 9 5481 |
| 31 | 6100 | 6721 | 7344 | 7969 | 8596 | 9225 | 9856 | 10 0489 | 10 1124 | 10 1761 |
| 32 | 10 2400 | 10 3041 | 10 3684 | 10 4329 | 10 4976 | 10 5625 | 10 6276 | 6929 | 7584 | 8241 |
| 33 | 8900 | 9561 | 11 0224 | 11 0889 | 11 1556 | 11 2225 | 11 2896 | 11 3569 | 11 4244 | 11 4921 |
| 34 | 11 5600 | 11 6281 | 6964 | 7649 | 8336 | 9025 | 9716 | 12 0409 | 12 1104 | 12 1801 |
| 35 | 12 2500 | 12 3201 | 12 3904 | 12 4609 | 12 5316 | 12 6025 | 12 6736 | 7449 | 8164 | 8881 |
| 36 | 9600 | 13 0321 | 13 1044 | 13 1769 | 13 2496 | 13 2225 | 13 3956 | 13 4689 | 13 5424 | 13 6161 |
| 37 | 13 6900 | 7641 | 8384 | 9129 | 9876 | 14 0625 | 14 1376 | 14 2129 | 14 2884 | 14 3641 |
| 38 | 14 4400 | 14 5161 | 14 5924 | 14 6689 | 14 7456 | 8225 | 8996 | 9769 | 15 0544 | 15 1321 |
| 39 | 15 2100 | 15 2881 | 15 3664 | 15 4449 | 15 5236 | 15 6025 | 15 6816 | 15 7609 | 8404 | 9201 |
| 40 | 16 0000 | 16 0801 | 16 1604 | 16 2409 | 16 3216 | 16 4025 | 16 4836 | 16 5649 | 16 6464 | 16 7281 |
| 41 | 8100 | 8921 | 9744 | 17 0569 | 17 1396 | 17 2225 | 17 3056 | 17 3889 | 17 4724 | 17 5561 |
| 42 | 17 6400 | 17 7241 | 17 8084 | 8929 | 9776 | 18 0625 | 18 1476 | 18 2329 | 18 3184 | 18 4041 |
| 43 | 18 4900 | 18 5761 | 18 6624 | 18 7489 | 18 8356 | 9225 | 19 0096 | 19 0969 | 19 1844 | 19 2721 |
| 44 | 19 3600 | 19 4481 | 19 5364 | 19 6249 | 19 7136 | 19 8025 | 8916 | 9809 | 20 0704 | 20 1601 |
| 45 | 20 2500 | 20 3401 | 20 4304 | 20 5209 | 20 6116 | 20 7025 | 20 7936 | 20 8849 | 9764 | 21 0681 |
| 46 | 21 1600 | 21 2521 | 21 3444 | 21 4369 | 21 5296 | 21 6225 | 21 7156 | 21 8089 | 21 9024 | 9961 |
| 47 | 22 0900 | 22 1841 | 22 2784 | 22 3729 | 22 4676 | 22 5625 | 22 6576 | 22 7529 | 22 8484 | 22 9441 |
| 48 | 23 0400 | 23 1361 | 23 2324 | 23 3289 | 23 4256 | 23 5225 | 23 6196 | 23 7169 | 23 8144 | 23 9121 |
| 49 | 24 0100 | 24 1081 | 24 2064 | 24 3049 | 24 4036 | 24 5025 | 24 6016 | 24 7009 | 24 8004 | 24 9001 |
| 50 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

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* From Jones' "Logarithmic Tables."

## TABLE III.—SQUARES—*Continued*

| 50 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 0000 | 25 1001 | 25 2004 | 25 3009 | 25 4016 | 25 5025 | 25 6036 | 25 7049 | 25 8064 | 25 9081 |
| 51 | 26 0100 | 26 1121 | 26 2144 | 26 3169 | 26 4196 | 26 5225 | 26 6256 | 26 7289 | 26 8324 | 26 9361 |
| 52 | 27 0400 | 27 1441 | 27 2484 | 27 3529 | 27 4576 | 27 5625 | 27 6676 | 27 7729 | 27 8784 | 27 9841 |
| 53 | 28 0900 | 28 1961 | 28 3024 | 28 4089 | 28 5156 | 28 6225 | 28 7296 | 28 8369 | 28 9444 | 29 0521 |
| 54 | 29 1600 | 29 2681 | 29 3764 | 29 4849 | 29 5936 | 29 7025 | 29 8116 | 29 9209 | 30 0304 | 30 1401 |
| 55 | 30 2500 | 30 3601 | 30 4704 | 30 5809 | 30 6916 | 30 8025 | 30 9136 | 31 0249 | 31 1364 | 31 2481 |
| 56 | 31 3600 | 31 4721 | 31 5844 | 31 6969 | 31 8096 | 31 9225 | 32 0356 | 32 1489 | 32 2624 | 32 3761 |
| 57 | 32 4900 | 32 6041 | 32 7184 | 32 8329 | 32 9476 | 33 0625 | 33 1776 | 33 2929 | 33 4084 | 33 5241 |
| 58 | 33 6400 | 33 7561 | 33 8724 | 33 9889 | 34 1056 | 34 2225 | 34 3396 | 34 4569 | 34 5744 | 34 6921 |
| 59 | 34 8100 | 34 9281 | 35 0464 | 35 1649 | 35 2836 | 35 4025 | 35 5216 | 35 6409 | 35 7604 | 35 8801 |
| 60 | 36 0000 | 36 1201 | 36 2404 | 36 3609 | 36 4816 | 36 6025 | 36 7236 | 36 8449 | 36 9664 | 37 0881 |
| 61 | 37 2100 | 37 3321 | 37 4544 | 37 5769 | 37 6996 | 37 8225 | 37 9456 | 38 0689 | 38 1924 | 38 3161 |
| 62 | 38 4400 | 38 5641 | 38 6884 | 38 8129 | 38 9376 | 39 0625 | 39 1876 | 39 3129 | 39 4384 | 39 5641 |
| 63 | 39 6900 | 39 8161 | 39 9424 | 40 0689 | 40 1956 | 40 3225 | 40 4496 | 40 5769 | 40 7044 | 40 8321 |
| 64 | 40 9600 | 41 0881 | 41 2164 | 41 3449 | 41 4736 | 41 6025 | 41 7316 | 41 8609 | 41 9904 | 42 1201 |
| 65 | 42 2500 | 42 3801 | 42 5104 | 42 6409 | 42 7716 | 42 9025 | 43 0336 | 43 1649 | 43 2964 | 43 4281 |
| 66 | 43 5600 | 43 6921 | 43 8244 | 43 9569 | 44 0896 | 44 2225 | 44 3556 | 44 4889 | 44 6224 | 44 7561 |
| 67 | 44 8900 | 45 0241 | 45 1584 | 45 2929 | 45 4276 | 45 5625 | 45 6976 | 45 8329 | 45 9684 | 46 1041 |
| 68 | 46 2400 | 46 3761 | 46 5124 | 46 6489 | 46 7856 | 46 9225 | 47 0596 | 47 1969 | 47 3344 | 47 4721 |
| 69 | 47 6100 | 47 7481 | 47 8864 | 48 0249 | 48 1636 | 48 3025 | 48 4416 | 48 5809 | 48 7204 | 48 8601 |
| 70 | 49 0000 | 49 1401 | 49 2804 | 49 4209 | 49 5616 | 49 7025 | 49 8436 | 49 9849 | 50 1264 | 50 2681 |
| 71 | 50 4100 | 50 5521 | 50 6944 | 50 8369 | 50 9796 | 51 1225 | 51 2656 | 51 4089 | 51 5524 | 51 6961 |
| 72 | 51 8400 | 51 9841 | 52 1284 | 52 2729 | 52 4176 | 52 5625 | 52 7076 | 52 8529 | 52 9984 | 53 1441 |
| 73 | 53 2900 | 53 4361 | 53 5824 | 53 7289 | 53 8756 | 54 0225 | 54 1696 | 54 3169 | 54 4644 | 54 6121 |
| 74 | 54 7600 | 54 9081 | 55 0564 | 55 2049 | 55 3536 | 55 5025 | 55 6516 | 55 8009 | 55 9504 | 56 1001 |
| 75 | 56 2500 | 56 4001 | 56 5504 | 56 7009 | 56 8516 | 57 0025 | 57 1536 | 57 3049 | 57 4564 | 57 6081 |
| 76 | 57 7600 | 57 9121 | 58 0644 | 58 2169 | 58 3696 | 58 5225 | 58 6756 | 58 8289 | 58 9824 | 59 1361 |
| 77 | 59 2900 | 59 4441 | 59 5984 | 59 7529 | 59 9076 | 60 0625 | 60 2176 | 60 3729 | 60 5284 | 60 6841 |
| 78 | 60 8400 | 60 9961 | 61 1524 | 61 3089 | 61 4656 | 61 6225 | 61 7796 | 61 9369 | 62 0944 | 62 2521 |
| 79 | 62 4100 | 62 5681 | 62 7264 | 62 8849 | 63 0436 | 63 2025 | 63 3616 | 63 5209 | 63 6804 | 63 8401 |
| 80 | 64 0000 | 64 1601 | 64 3204 | 64 4809 | 64 6416 | 64 8025 | 64 9636 | 65 1249 | 65 2864 | 65 4481 |
| 81 | 65 6100 | 65 7721 | 65 9344 | 66 0969 | 66 2596 | 66 4225 | 66 5856 | 66 7489 | 66 9124 | 67 0761 |
| 82 | 67 2400 | 67 4041 | 67 5684 | 67 7329 | 67 8976 | 68 0625 | 68 2276 | 68 3929 | 68 5584 | 68 7241 |
| 83 | 68 8900 | 69 0561 | 69 2224 | 69 3889 | 69 5556 | 69 7225 | 69 8896 | 70 0569 | 70 2244 | 70 3921 |
| 84 | 70 5600 | 70 7281 | 70 8964 | 71 0649 | 71 2336 | 71 4025 | 71 5716 | 71 7409 | 71 9104 | 72 0801 |
| 85 | 72 2500 | 72 4201 | 72 5904 | 72 7609 | 72 9316 | 73 1025 | 73 2736 | 73 4449 | 73 6164 | 73 7881 |
| 86 | 73 9600 | 74 1321 | 74 3044 | 74 4769 | 74 6496 | 74 8225 | 74 9956 | 75 1689 | 75 3424 | 75 5161 |
| 87 | 75 6900 | 75 8641 | 76 0384 | 76 2129 | 76 3876 | 76 5625 | 76 7376 | 76 9129 | 77 0884 | 77 2641 |
| 88 | 77 4400 | 77 6161 | 77 7924 | 77 9689 | 78 1456 | 78 3225 | 78 4996 | 78 6769 | 78 8544 | 79 0321 |
| 89 | 79 2100 | 79 3881 | 79 5664 | 79 7449 | 79 9236 | 80 1025 | 80 2816 | 80 4609 | 80 6404 | 80 8201 |
| 90 | 81 0000 | 81 1801 | 81 3604 | 81 5409 | 81 7216 | 81 9025 | 82 0836 | 82 2649 | 82 4464 | 82 6281 |
| 91 | 82 8100 | 82 9921 | 83 1744 | 83 3569 | 83 5396 | 83 7225 | 83 9056 | 84 0889 | 84 2724 | 84 4561 |
| 92 | 84 6400 | 84 8241 | 85 0084 | 85 1929 | 85 3776 | 85 5625 | 85 7476 | 85 9329 | 86 1184 | 86 3041 |
| 93 | 86 4900 | 86 6761 | 86 8624 | 87 0489 | 87 2356 | 87 4225 | 87 6096 | 87 7969 | 87 9844 | 88 1721 |
| 94 | 88 3600 | 88 5481 | 88 7364 | 88 9249 | 89 1136 | 89 3025 | 89 4916 | 89 6809 | 89 8704 | 90 0601 |
| 95 | 90 2500 | 90 4401 | 90 6304 | 90 8209 | 91 0116 | 91 2025 | 91 3936 | 91 5849 | 91 7764 | 91 9681 |
| 96 | 92 1600 | 92 3521 | 92 5444 | 92 7369 | 92 9296 | 93 1225 | 93 3156 | 93 5089 | 93 7024 | 93 8961 |
| 97 | 94 0900 | 94 2841 | 94 4784 | 94 6729 | 94 8676 | 95 0625 | 95 2576 | 95 4529 | 95 6484 | 95 8441 |
| 98 | 96 0400 | 96 2361 | 96 4324 | 96 6289 | 96 8256 | 97 0225 | 97 2196 | 97 4169 | 97 6144 | 97 8121 |
| 99 | 98 0100 | 98 2081 | 98 4064 | 98 6049 | 98 8036 | 99 0025 | 99 2016 | 99 4009 | 99 6004 | 99 8001 |
| 100 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

## TABLE IV.—NATURAL SINES

| | 0′ | 6′ | 12′ | 18′ | 24′ | 30′ | 36′ | 42′ | 48′ | 54′ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0000 | 0017 | 0035 | 0052 | 0070 | 0087 | 0105 | 0122 | 0140 | 0157 | 3 | 6 | 9 | 12 | 15 |
| 1 | 0175 | 0192 | 0209 | 0227 | 0244 | 0262 | 0279 | 0297 | 0314 | 0332 | 3 | 6 | 9 | 12 | 15 |
| 2 | 0349 | 0366 | 0384 | 0401 | 0419 | 0436 | 0454 | 0471 | 0488 | 0506 | 3 | 6 | 9 | 12 | 15 |
| 3 | 0523 | 0541 | 0558 | 0576 | 0593 | 0610 | 0628 | 0645 | 0663 | 0680 | 3 | 6 | 9 | 12 | 15 |
| 4 | 0698 | 0715 | 0732 | 0750 | 0767 | 0785 | 0802 | 0819 | 0837 | 0854 | 3 | 6 | 9 | 12 | 15 |
| 5 | 0872 | 0889 | 0906 | 0924 | 0941 | 0958 | 0976 | 0993 | 1011 | 1028 | 3 | 6 | 9 | 12 | 14 |
| 6 | 1045 | 1063 | 1080 | 1097 | 1115 | 1132 | 1149 | 1167 | 1184 | 1201 | 3 | 6 | 9 | 12 | 14 |
| 7 | 1219 | 1236 | 1253 | 1271 | 1288 | 1305 | 1323 | 1340 | 1357 | 1374 | 3 | 6 | 9 | 12 | 14 |
| 8 | 1392 | 1409 | 1426 | 1444 | 1461 | 1478 | 1495 | 1513 | 1530 | 1547 | 3 | 6 | 9 | 12 | 14 |
| 9 | 1564 | 1582 | 1599 | 1616 | 1633 | 1650 | 1668 | 1685 | 1702 | 1719 | 3 | 6 | 9 | 12 | 14 |
| 10 | 1736 | 1754 | 1771 | 1788 | 1805 | 1822 | 1840 | 1857 | 1874 | 1891 | 3 | 6 | 9 | 12 | 14 |
| 11 | 1908 | 1925 | 1942 | 1959 | 1977 | 1994 | 2011 | 2028 | 2045 | 2062 | 3 | 6 | 9 | 11 | 14 |
| 12 | 2079 | 2096 | 2113 | 2130 | 2147 | 2164 | 2181 | 2198 | 2215 | 2232 | 3 | 6 | 9 | 11 | 14 |
| 13 | 2250 | 2267 | 2284 | 2300 | 2317 | 2334 | 2351 | 2368 | 2385 | 2402 | 3 | 6 | 8 | 11 | 14 |
| 14 | 2419 | 2436 | 2453 | 2470 | 2487 | 2504 | 2521 | 2538 | 2554 | 2571 | 3 | 6 | 8 | 11 | 14 |
| 15 | 2588 | 2605 | 2622 | 2639 | 2656 | 2672 | 2689 | 2706 | 2723 | 2740 | 3 | 6 | 8 | 11 | 14 |
| 16 | 2756 | 2773 | 2790 | 2807 | 2823 | 2840 | 2857 | 2874 | 2890 | 2907 | 3 | 6 | 8 | 11 | 14 |
| 17 | 2924 | 2940 | 2957 | 2974 | 2990 | 3007 | 3024 | 3040 | 3057 | 3074 | 3 | 6 | 8 | 11 | 14 |
| 18 | 3090 | 3107 | 3123 | 3140 | 3156 | 3173 | 3190 | 3206 | 3223 | 3239 | 3 | 6 | 8 | 11 | 14 |
| 19 | 3256 | 3272 | 3289 | 3305 | 3322 | 3338 | 3355 | 3371 | 3387 | 3404 | 3 | 5 | 8 | 11 | 14 |
| 20 | 3420 | 3437 | 3453 | 3469 | 3486 | 3502 | 3518 | 3535 | 3551 | 3567 | 3 | 5 | 8 | 11 | 14 |
| 21 | 3584 | 3600 | 3616 | 3633 | 3649 | 3665 | 3681 | 3697 | 3714 | 3730 | 3 | 5 | 8 | 11 | 14 |
| 22 | 3746 | 3762 | 3778 | 3795 | 3811 | 3827 | 3843 | 3859 | 3875 | 3891 | 3 | 5 | 8 | 11 | 14 |
| 23 | 3907 | 3923 | 3939 | 3955 | 3971 | 3987 | 4003 | 4019 | 4035 | 4051 | 3 | 5 | 8 | 11 | 14 |
| 24 | 4067 | 4083 | 4099 | 4115 | 4131 | 4147 | 4163 | 4179 | 4195 | 4210 | 3 | 5 | 8 | 11 | 13 |
| 25 | 4226 | 4242 | 4258 | 4274 | 4289 | 4305 | 4321 | 4337 | 4352 | 4368 | 3 | 5 | 8 | 11 | 13 |
| 26 | 4384 | 4399 | 4415 | 4431 | 4446 | 4462 | 4478 | 4493 | 4509 | 4524 | 3 | 5 | 8 | 10 | 13 |
| 27 | 4540 | 4555 | 4571 | 4586 | 4602 | 4617 | 4633 | 4648 | 4664 | 4679 | 3 | 5 | 8 | 10 | 13 |
| 28 | 4695 | 4710 | 4726 | 4741 | 4756 | 4772 | 4787 | 4802 | 4818 | 4833 | 3 | 5 | 8 | 10 | 13 |
| 29 | 4848 | 4863 | 4879 | 4894 | 4909 | 4924 | 4939 | 4955 | 4970 | 4985 | 3 | 5 | 8 | 10 | 13 |
| 30 | 5000 | 5015 | 5030 | 5045 | 5060 | 5075 | 5090 | 5105 | 5120 | 5135 | 3 | 5 | 8 | 10 | 13 |
| 31 | 5150 | 5165 | 5180 | 5195 | 5210 | 5225 | 5240 | 5255 | 5270 | 5284 | 2 | 5 | 7 | 10 | 12 |
| 32 | 5299 | 5314 | 5329 | 5344 | 5358 | 5373 | 5388 | 5402 | 5417 | 5432 | 2 | 5 | 7 | 10 | 12 |
| 33 | 5446 | 5461 | 5476 | 5490 | 5505 | 5519 | 5534 | 5548 | 5563 | 5577 | 2 | 5 | 7 | 10 | 12 |
| 34 | 5592 | 5606 | 5621 | 5635 | 5650 | 5664 | 5678 | 5693 | 5707 | 5721 | 2 | 5 | 7 | 10 | 12 |
| 35 | 5736 | 5750 | 5764 | 5779 | 5793 | 5807 | 5821 | 5835 | 5850 | 5864 | 2 | 5 | 7 | 10 | 12 |
| 36 | 5878 | 5892 | 5906 | 5920 | 5934 | 5948 | 5962 | 5976 | 5990 | 6004 | 2 | 5 | 7 | 9 | 12 |
| 37 | 6018 | 6032 | 6046 | 6060 | 6074 | 6088 | 6101 | 6115 | 6129 | 6143 | 2 | 5 | 7 | 9 | 12 |
| 38 | 6157 | 6170 | 6184 | 6198 | 6211 | 6225 | 6239 | 6252 | 6266 | 6280 | 2 | 5 | 7 | 9 | 11 |
| 39 | 6293 | 6307 | 6320 | 6334 | 6347 | 6361 | 6374 | 6388 | 6401 | 6414 | 2 | 4 | 7 | 9 | 11 |
| 40 | 6428 | 6441 | 6455 | 6468 | 6481 | 6494 | 6508 | 6521 | 6534 | 6547 | 2 | 4 | 7 | 9 | 11 |
| 41 | 6561 | 6574 | 6587 | 6600 | 6613 | 6626 | 6639 | 6652 | 6665 | 6678 | 2 | 4 | 7 | 9 | 11 |
| 42 | 6691 | 6704 | 6717 | 6730 | 6743 | 6756 | 6769 | 6782 | 6794 | 6807 | 2 | 4 | 6 | 9 | 11 |
| 43 | 6820 | 6833 | 6845 | 6858 | 6871 | 6884 | 6896 | 6909 | 6921 | 6934 | 2 | 4 | 6 | 8 | 11 |
| 44 | 6947 | 6959 | 6972 | 6984 | 6997 | 7009 | 7022 | 7034 | 7046 | 7059 | 2 | 4 | 6 | 8 | 10 |

Table IV.—Natural Sines—*Continued*

| | 0′ | 6′ | 12′ | 18′ | 24′ | 30′ | 36′ | 42′ | 48′ | 54′ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45° | 7071 | 7083 | 7096 | 7108 | 7120 | 7133 | 7145 | 7157 | 7169 | 7181 | 2 | 4 | 6 | 8 | 10 |
| 46 | 7193 | 7206 | 7218 | 7230 | 7242 | 7254 | 7266 | 7278 | 7290 | 7302 | 2 | 4 | 6 | 8 | 10 |
| 47 | 7314 | 7325 | 7337 | 7349 | 7361 | 7373 | 7385 | 7396 | 7408 | 7420 | 2 | 4 | 6 | 8 | 10 |
| 48 | 7431 | 7443 | 7455 | 7466 | 7478 | 7490 | 7501 | 7513 | 7524 | 7536 | 2 | 4 | 6 | 8 | 10 |
| 49 | 7547 | 7558 | 7570 | 7581 | 7593 | 7604 | 7615 | 7627 | 7638 | 7649 | 2 | 4 | 6 | 8 | 9 |
| 50 | 7660 | 7672 | 7683 | 7694 | 7705 | 7716 | 7727 | 7738 | 7749 | 7760 | 2 | 4 | 6 | 7 | 9 |
| 51 | 7771 | 7782 | 7793 | 7804 | 7815 | 7826 | 7837 | 7848 | 7859 | 7869 | 2 | 4 | 5 | 7 | 9 |
| 52 | 7880 | 7891 | 7902 | 7912 | 7923 | 7934 | 7944 | 7955 | 7965 | 7976 | 2 | 4 | 5 | 7 | 9 |
| 53 | 7986 | 7997 | 8007 | 8018 | 8028 | 8039 | 8049 | 8059 | 8070 | 8080 | 2 | 3 | 5 | 7 | 9 |
| 54 | 8090 | 8100 | 8111 | 8121 | 8131 | 8141 | 8151 | 8161 | 8171 | 8181 | 2 | 3 | 5 | 7 | 8 |
| 55 | 8192 | 8202 | 8211 | 8221 | 8231 | 8241 | 8251 | 8261 | 8271 | 8281 | 2 | 3 | 5 | 7 | 8 |
| 56 | 8290 | 8300 | 8310 | 8320 | 8329 | 8339 | 8348 | 8358 | 8368 | 8377 | 2 | 3 | 5 | 6 | 8 |
| 57 | 8387 | 8396 | 8406 | 8415 | 8425 | 8434 | 8443 | 8453 | 8462 | 8471 | 2 | 3 | 5 | 6 | 8 |
| 58 | 8480 | 8490 | 8499 | 8508 | 8517 | 8526 | 8536 | 8545 | 8554 | 8563 | 2 | 3 | 5 | 6 | 8 |
| 59 | 8572 | 8581 | 8590 | 8599 | 8607 | 8616 | 8625 | 8634 | 8643 | 8652 | 1 | 3 | 4 | 6 | 7 |
| 60 | 8660 | 8669 | 8678 | 8686 | 8695 | 8704 | 8712 | 8721 | 8729 | 8738 | 1 | 3 | 4 | 6 | 7 |
| 61 | 8746 | 8755 | 8763 | 8771 | 8780 | 8788 | 8796 | 8805 | 8813 | 8821 | 1 | 3 | 4 | 6 | 7 |
| 62 | 8829 | 8838 | 8846 | 8854 | 8862 | 8870 | 8878 | 8886 | 8894 | 8902 | 1 | 3 | 4 | 5 | 7 |
| 63 | 8910 | 8918 | 8926 | 8934 | 8942 | 8949 | 8957 | 8965 | 8973 | 8980 | 1 | 3 | 4 | 5 | 6 |
| 64 | 8988 | 8996 | 9003 | 9011 | 9018 | 9026 | 9033 | 9041 | 9048 | 9056 | 1 | 3 | 4 | 5 | 6 |
| 65 | 9063 | 9070 | 9078 | 9085 | 9092 | 9100 | 9107 | 9114 | 9121 | 9128 | 1 | 2 | 4 | 5 | 6 |
| 66 | 9135 | 9143 | 9150 | 9157 | 9164 | 9171 | 9178 | 9184 | 9191 | 9198 | 1 | 2 | 3 | 4 | 6 |
| 67 | 9205 | 9212 | 9219 | 9225 | 9232 | 9239 | 9245 | 9252 | 9259 | 9265 | 1 | 2 | 3 | 4 | 6 |
| 68 | 9272 | 9278 | 9285 | 9291 | 9298 | 9304 | 9311 | 9317 | 9323 | 9330 | 1 | 2 | 3 | 4 | 5 |
| 69 | 9336 | 9342 | 9348 | 9354 | 9361 | 9367 | 9373 | 9379 | 9385 | 9391 | 1 | 2 | 3 | 4 | 5 |
| 70 | 9397 | 9403 | 9409 | 9415 | 9421 | 9426 | 9432 | 9438 | 9444 | 9449 | 1 | 2 | 3 | 4 | 5 |
| 71 | 9455 | 9461 | 9466 | 9472 | 9478 | 9483 | 9489 | 9494 | 9500 | 9505 | 1 | 2 | 3 | 4 | 5 |
| 72 | 9511 | 9516 | 9521 | 9527 | 9532 | 9537 | 9542 | 9548 | 9553 | 9558 | 1 | 2 | 3 | 4 | 4 |
| 73 | 9563 | 9568 | 9573 | 9578 | 9583 | 9588 | 9593 | 9598 | 9603 | 9608 | 1 | 2 | 2 | 3 | 4 |
| 74 | 9613 | 9617 | 9622 | 9627 | 9632 | 9636 | 9641 | 9646 | 9650 | 9655 | 1 | 2 | 2 | 3 | 4 |
| 75 | 9659 | 9664 | 9668 | 9673 | 9677 | 9681 | 9686 | 9690 | 9694 | 9699 | 1 | 1 | 2 | 3 | 4 |
| 76 | 9703 | 9707 | 9711 | 9715 | 9720 | 9724 | 9728 | 9732 | 9736 | 9740 | 1 | 1 | 2 | 3 | 3 |
| 77 | 9744 | 9748 | 9751 | 9755 | 9759 | 9763 | 9767 | 9770 | 9774 | 9778 | 1 | 1 | 2 | 3 | 3 |
| 78 | 9781 | 9785 | 9789 | 9792 | 9796 | 9799 | 9803 | 9806 | 9810 | 9813 | 1 | 1 | 2 | 2 | 3 |
| 79 | 9816 | 9820 | 9823 | 9826 | 9829 | 9833 | 9836 | 9839 | 9842 | 9845 | 1 | 1 | 2 | 2 | 3 |
| 80 | 9848 | 9851 | 9854 | 9857 | 9860 | 9863 | 9866 | 9869 | 9871 | 9874 | 0 | 1 | 1 | 2 | 2 |
| 81 | 9877 | 9880 | 9882 | 9885 | 9888 | 9890 | 9893 | 9895 | 9898 | 9900 | 0 | 1 | 1 | 2 | 2 |
| 82 | 9903 | 9905 | 9907 | 9910 | 9912 | 9914 | 9917 | 9919 | 9921 | 9923 | 0 | 1 | 1 | 2 | 2 |
| 83 | 9925 | 9928 | 9930 | 9932 | 9934 | 9936 | 9938 | 9940 | 9942 | 9943 | 0 | 1 | 1 | 1 | 2 |
| 84 | 9945 | 9947 | 9949 | 9951 | 9952 | 9954 | 9956 | 9957 | 9959 | 9960 | 0 | 1 | 1 | 1 | 1 |
| 85 | 9962 | 9963 | 9965 | 9966 | 9968 | 9969 | 9971 | 9972 | 9973 | 9974 | 0 | 0 | 1 | 1 | 1 |
| 86 | 9976 | 9977 | 9978 | 9979 | 9980 | 9981 | 9982 | 9983 | 9984 | 9985 | 0 | 0 | 1 | 1 | 1 |
| 87 | 9986 | 9987 | 9988 | 9989 | 9990 | 9990 | 9991 | 9992 | 9993 | 9993 | 0 | 0 | 0 | 1 | 1 |
| 88 | 9994 | 9995 | 9995 | 9996 | 9996 | 9997 | 9997 | 9997 | 9998 | 9988 | 0 | 0 | 0 | 0 | 0 |
| 89 | 9998 | 9999 | 9999 | 9999 | 9999 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | nearly | nearly | nearly | nearly | nearly | | | | | |

### TABLE V.—NATURAL COSINES

| | 0′ | 6′ | 12′ | 18′ | 24′ | 30′ | 36′ | 42′ | 48′ | 54′ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 9999 | 9999 | 9999 | 9999 | 0 | 0 | 0 | 0 | 0 |
| 1 | 9998 | 9998 | 9998 | 9997 | 9997 | 9997 | 9996 | 9996 | 9995 | 9995 | 0 | 0 | 0 | 0 | 0 |
| 2 | 9994 | 9993 | 9993 | 9992 | 9991 | 9990 | 9990 | 9989 | 9988 | 9987 | 0 | 0 | 0 | 1 | 1 |
| 3 | 9986 | 9985 | 9984 | 9983 | 9982 | 9981 | 9980 | 9979 | 9978 | 9977 | 0 | 0 | 1 | 1 | 1 |
| 4 | 9976 | 9974 | 9973 | 9972 | 9971 | 9969 | 9968 | 9966 | 9965 | 9963 | 0 | 0 | 1 | 1 | 1 |
| 5 | 9962 | 9960 | 9959 | 9957 | 9956 | 9954 | 9952 | 9951 | 9949 | 9947 | 0 | 0 | 1 | 1 | 2 |
| 6 | 9945 | 9943 | 9942 | 9940 | 9938 | 9936 | 9934 | 9932 | 9930 | 9928 | 0 | 1 | 1 | 1 | 2 |
| 7 | 9925 | 9923 | 9921 | 9919 | 9917 | 9914 | 9912 | 9910 | 9907 | 9905 | 0 | 1 | 1 | 2 | 2 |
| 8 | 9903 | 9900 | 9898 | 9895 | 9893 | 9890 | 9888 | 9885 | 9882 | 9880 | 0 | 1 | 1 | 2 | 2 |
| 9 | 9877 | 9874 | 9871 | 9869 | 9866 | 9863 | 9860 | 9857 | 9854 | 9851 | 0 | 1 | 1 | 2 | 2 |
| 10 | 9848 | 9845 | 9842 | 9839 | 9836 | 9833 | 9829 | 9826 | 9823 | 9820 | 1 | 1 | 2 | 2 | 3 |
| 11 | 9816 | 9813 | 9810 | 9806 | 9803 | 9799 | 9796 | 9792 | 9789 | 9785 | 1 | 1 | 2 | 2 | 3 |
| 12 | 9781 | 9778 | 9774 | 9770 | 9767 | 9763 | 9759 | 9755 | 9751 | 9748 | 1 | 1 | 2 | 3 | 3 |
| 13 | 9744 | 9740 | 9736 | 9732 | 9728 | 9724 | 9720 | 9715 | 9711 | 9707 | 1 | 1 | 2 | 3 | 3 |
| 14 | 9703 | 9699 | 9694 | 9690 | 9686 | 9681 | 9677 | 9673 | 9668 | 9664 | 1 | 1 | 2 | 3 | 4 |
| 15 | 9659 | 9655 | 9650 | 9646 | 9641 | 9636 | 9632 | 9627 | 9622 | 9617 | 1 | 2 | 2 | 3 | 4 |
| 16 | 9613 | 9608 | 9603 | 9598 | 9593 | 9588 | 9583 | 9578 | 9573 | 9568 | 1 | 2 | 2 | 3 | 4 |
| 17 | 9563 | 9558 | 9553 | 9548 | 9542 | 9537 | 9532 | 9527 | 9521 | 9516 | 1 | 2 | 3 | 4 | 4 |
| 18 | 9511 | 9505 | 9500 | 9494 | 9489 | 9483 | 9478 | 9472 | 9466 | 9461 | 1 | 2 | 3 | 4 | 5 |
| 19 | 9455 | 9449 | 9444 | 9438 | 9432 | 9426 | 9421 | 9415 | 9409 | 9403 | 1 | 2 | 3 | 4 | 5 |
| 20 | 9397 | 9391 | 9385 | 9379 | 9373 | 9367 | 9361 | 9354 | 9348 | 9342 | 1 | 2 | 3 | 4 | 5 |
| 21 | 9336 | 9330 | 9323 | 9317 | 9311 | 9304 | 9298 | 9291 | 9285 | 9278 | 1 | 2 | 3 | 4 | 5 |
| 22 | 9272 | 9265 | 9259 | 9252 | 9245 | 9239 | 9232 | 9225 | 9219 | 9212 | 1 | 2 | 3 | 4 | 6 |
| 23 | 9205 | 9198 | 9191 | 9184 | 9178 | 9171 | 9164 | 9157 | 9150 | 9143 | 1 | 2 | 3 | 5 | 6 |
| 24 | 9135 | 9128 | 9121 | 9114 | 9107 | 9100 | 9092 | 9085 | 9078 | 9070 | 1 | 2 | 4 | 5 | 6 |
| 25 | 9063 | 9056 | 9048 | 9041 | 9033 | 9026 | 9018 | 9011 | 9003 | 8996 | 1 | 3 | 4 | 5 | 6 |
| 26 | 8988 | 8980 | 8973 | 8965 | 8957 | 8949 | 8942 | 8934 | 8926 | 8918 | 1 | 3 | 4 | 5 | 6 |
| 27 | 8910 | 8902 | 8894 | 8886 | 8878 | 8870 | 8862 | 8854 | 8846 | 8838 | 1 | 3 | 4 | 5 | 7 |
| 28 | 8829 | 8821 | 8813 | 8805 | 8796 | 8788 | 8780 | 8771 | 8763 | 8755 | 1 | 3 | 4 | 6 | 7 |
| 29 | 8746 | 8738 | 8729 | 8721 | 8712 | 8704 | 8695 | 8686 | 8678 | 8669 | 1 | 3 | 4 | 6 | 7 |
| 30 | 8660 | 8652 | 8643 | 8634 | 8625 | 8616 | 8607 | 8599 | 8590 | 8581 | 1 | 3 | 4 | 6 | 7 |
| 31 | 8572 | 8563 | 8554 | 8545 | 8536 | 8526 | 8517 | 8508 | 8499 | 8490 | 2 | 3 | 5 | 6 | 8 |
| 32 | 8480 | 8471 | 8462 | 8453 | 8443 | 8434 | 8425 | 8415 | 8406 | 8396 | 2 | 3 | 5 | 6 | 8 |
| 33 | 8387 | 8377 | 8368 | 8358 | 8348 | 8339 | 8329 | 8320 | 8310 | 8300 | 2 | 3 | 5 | 6 | 8 |
| 34 | 8290 | 8281 | 8271 | 8261 | 8251 | 8241 | 8231 | 8221 | 8211 | 8202 | 2 | 3 | 5 | 7 | 8 |
| 35 | 8192 | 8181 | 8171 | 8161 | 8151 | 8141 | 8131 | 8121 | 8111 | 8100 | 2 | 3 | 5 | 7 | 8 |
| 36 | 8090 | 8080 | 8070 | 8059 | 8049 | 8039 | 8028 | 8018 | 8007 | 7997 | 2 | 3 | 5 | 7 | 9 |
| 37 | 7986 | 7976 | 7965 | 7955 | 7944 | 7934 | 7923 | 7912 | 7902 | 7891 | 2 | 4 | 5 | 7 | 9 |
| 38 | 7880 | 7869 | 7859 | 7848 | 7837 | 7826 | 7815 | 7804 | 7793 | 7782 | 2 | 4 | 5 | 7 | 9 |
| 39 | 7771 | 7760 | 7749 | 7738 | 7727 | 7716 | 7705 | 7694 | 7683 | 7672 | 2 | 4 | 6 | 7 | 9 |
| 40 | 7660 | 7649 | 7638 | 7627 | 7615 | 7604 | 7593 | 7581 | 7570 | 7559 | 2 | 4 | 6 | 8 | 9 |
| 41 | 7547 | 7536 | 7524 | 7513 | 7501 | 7490 | 7478 | 7466 | 7455 | 7443 | 2 | 4 | 6 | 8 | 10 |
| 42 | 7431 | 7420 | 7408 | 7396 | 7385 | 7373 | 7361 | 7349 | 7337 | 7325 | 2 | 4 | 6 | 8 | 10 |
| 43 | 7314 | 7302 | 7290 | 7278 | 7266 | 7254 | 7242 | 7230 | 7218 | 7206 | 2 | 4 | 6 | 8 | 10 |
| 44 | 7193 | 7181 | 7169 | 7157 | 7145 | 7133 | 7120 | 7108 | 7096 | 7083 | 2 | 4 | 6 | 8 | 10 |

*Note.*—Numbers in difference columns to be subtracted, not added.

Table V.—Natural Cosines—*Continued*

| | 0′ | 6′ | 12′ | 18′ | 24′ | 30′ | 36′ | 42′ | 48′ | 54′ | 1 2 3 | 4 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45° | 7071 | 7059 | 7046 | 7034 | 7022 | 7009 | 6997 | 6984 | 6972 | 6959 | 2 4 6 | 8 10 |
| 46 | 6947 | 6934 | 6921 | 6909 | 6896 | 6884 | 6871 | 6858 | 6845 | 6833 | 2 4 6 | 8 11 |
| 47 | 6820 | 6807 | 6794 | 6782 | 6769 | 6756 | 6743 | 6730 | 6717 | 6704 | 2 4 6 | 9 11 |
| 48 | 6691 | 6678 | 6655 | 6652 | 6639 | 6626 | 6613 | 6600 | 6587 | 6574 | 2 4 7 | 9 11 |
| 49 | 6561 | 6547 | 6534 | 6521 | 6508 | 6494 | 6481 | 6468 | 6455 | 6441 | 2 4 7 | 9 11 |
| 50 | 6428 | 6414 | 6401 | 6388 | 6374 | 6361 | 6347 | 6334 | 6320 | 6307 | 2 4 7 | 9 11 |
| 51 | 6293 | 6280 | 6266 | 6252 | 6239 | 6225 | 6211 | 6198 | 6184 | 6170 | 2 5 7 | 9 11 |
| 52 | 6157 | 6143 | 6129 | 6115 | 6101 | 6088 | 6074 | 6060 | 6046 | 6032 | 2 5 7 | 9 12 |
| 53 | 6018 | 6004 | 5990 | 5976 | 5962 | 5948 | 5934 | 5920 | 5906 | 5892 | 2 5 7 | 9 12 |
| 54 | 5878 | 5864 | 5850 | 5835 | 5821 | 5807 | 5793 | 5779 | 5764 | 5750 | 2 5 7 | 9 12 |
| 55 | 5736 | 5721 | 5707 | 5693 | 5678 | 5664 | 5650 | 5635 | 5621 | 5606 | 2 5 7 | 10 12 |
| 56 | 5592 | 5577 | 5563 | 5548 | 5534 | 5519 | 5505 | 5490 | 5476 | 5461 | 2 5 7 | 10 12 |
| 57 | 5446 | 5432 | 5417 | 5402 | 5388 | 5373 | 5358 | 5344 | 5329 | 5314 | 2 5 7 | 10 12 |
| 58 | 5299 | 5284 | 5270 | 5255 | 5240 | 5225 | 5210 | 5195 | 5180 | 5165 | 2 5 7 | 10 12 |
| 59 | 5150 | 5135 | 5120 | 5105 | 5090 | 5075 | 5060 | 5045 | 5030 | 5015 | 3 5 8 | 10 13 |
| 60 | 5000 | 4985 | 4970 | 4955 | 4939 | 4924 | 4909 | 4894 | 4879 | 4863 | 3 5 8 | 10 13 |
| 61 | 4848 | 4833 | 4818 | 4802 | 4787 | 4772 | 4756 | 4741 | 4726 | 4710 | 3 5 8 | 10 13 |
| 62 | 4695 | 4679 | 4664 | 4648 | 4633 | 4617 | 4602 | 4586 | 4571 | 4555 | 3 5 8 | 10 13 |
| 63 | 4540 | 4524 | 4509 | 4493 | 4478 | 4462 | 4446 | 4431 | 4415 | 4399 | 3 5 8 | 10 13 |
| 64 | 4384 | 4368 | 4352 | 4337 | 4321 | 4305 | 4289 | 4274 | 4258 | 4242 | 3 5 8 | 11 13 |
| 65 | 4226 | 4210 | 4195 | 4179 | 4163 | 4147 | 4131 | 4115 | 4099 | 4083 | 3 5 8 | 11 13 |
| 66 | 4067 | 4051 | 4035 | 4019 | 4003 | 3987 | 3971 | 3955 | 3939 | 3923 | 3 5 8 | 11 14 |
| 67 | 3907 | 3891 | 3875 | 3859 | 3843 | 3827 | 3811 | 3795 | 3778 | 3762 | 3 5 8 | 11 14 |
| 68 | 3746 | 3730 | 3714 | 3697 | 3681 | 3665 | 3649 | 3633 | 3616 | 3600 | 3 5 8 | 11 14 |
| 69 | 3584 | 3567 | 3551 | 3535 | 3518 | 3502 | 3486 | 3469 | 3453 | 3437 | 3 5 8 | 11 14 |
| 70 | 3420 | 3404 | 3387 | 3371 | 3355 | 3338 | 3322 | 3305 | 3289 | 3272 | 3 5 8 | 11 14 |
| 71 | 3256 | 3239 | 3223 | 3206 | 3190 | 3173 | 3156 | 3140 | 3123 | 3107 | 3 6 8 | 11 14 |
| 72 | 3090 | 3074 | 3057 | 3040 | 3024 | 3007 | 2990 | 2974 | 2957 | 2940 | 3 6 8 | 11 14 |
| 73 | 2924 | 2907 | 2890 | 2874 | 2857 | 2840 | 2823 | 2807 | 2790 | 2773 | 3 6 8 | 11 14 |
| 74 | 2756 | 2740 | 2723 | 2706 | 2689 | 2672 | 2656 | 2639 | 2622 | 2605 | 3 6 8 | 11 14 |
| 75 | 2588 | 2571 | 2554 | 2538 | 2521 | 2504 | 2487 | 2470 | 2453 | 2436 | 3 6 8 | 11 14 |
| 76 | 2419 | 2402 | 2385 | 2368 | 2351 | 2334 | 2317 | 2300 | 2284 | 2267 | 3 6 8 | 11 14 |
| 77 | 2250 | 2233 | 2215 | 2198 | 2181 | 2164 | 2147 | 2130 | 2113 | 2096 | 3 6 9 | 11 14 |
| 78 | 2079 | 2062 | 2045 | 2028 | 2011 | 1994 | 1977 | 1959 | 1942 | 1925 | 3 6 9 | 11 14 |
| 79 | 1908 | 1891 | 1874 | 1857 | 1840 | 1822 | 1805 | 1788 | 1771 | 1754 | 3 6 9 | 12 14 |
| 80 | 1736 | 1719 | 1702 | 1685 | 1668 | 1650 | 1633 | 1616 | 1599 | 1582 | 3 6 9 | 12 14 |
| 81 | 1564 | 1547 | 1530 | 1513 | 1495 | 1478 | 1461 | 1444 | 1426 | 1409 | 3 6 9 | 12 14 |
| 82 | 1392 | 1374 | 1357 | 1340 | 1323 | 1305 | 1288 | 1271 | 1253 | 1236 | 3 6 9 | 12 14 |
| 83 | 1219 | 1201 | 1184 | 1167 | 1149 | 1132 | 1115 | 1097 | 1080 | 1063 | 3 6 9 | 12 14 |
| 84 | 1045 | 1028 | 1011 | 0993 | 0976 | 0958 | 0941 | 0924 | 0906 | 0889 | 3 6 9 | 12 14 |
| 85 | 0872 | 0854 | 0837 | 0819 | 0802 | 0785 | 0767 | 0750 | 0732 | 0715 | 3 6 9 | 12 15 |
| 86 | 0698 | 0680 | 0663 | 0645 | 0628 | 0610 | 0593 | 0576 | 0558 | 0541 | 3 6 9 | 12 15 |
| 87 | 0523 | 0506 | 0488 | 0471 | 0454 | 0436 | 0419 | 0401 | 0384 | 0366 | 3 6 9 | 12 15 |
| 88 | 0349 | 0332 | 0314 | 0297 | 0279 | 0262 | 0244 | 0227 | 0209 | 0192 | 3 6 9 | 12 15 |
| 89 | 0175 | 0157 | 0140 | 0122 | 0105 | 0087 | 0070 | 0052 | 0035 | 0017 | 3 6 9 | 12 15 |

*Note.*—Numbers in difference columns to be subtracted, not added.

### TABLE VI.—NATURAL TANGENTS

| | 0′ | 6′ | 12′ | 18′ | 24′ | 30′ | 36′ | 42′ | 48′ | 54′ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0° | .0000 | 0017 | 0035 | 0052 | 0070 | 0087 | 0105 | 0122 | 0140 | 0157 | 3 | 6 | 9 | 12 | 14 |
| 1 | .0175 | 0192 | 0209 | 0227 | 0244 | 0262 | 0279 | 0297 | 0314 | 0332 | 3 | 6 | 9 | 12 | 15 |
| 2 | .0349 | 0367 | 0384 | 0402 | 0419 | 0437 | 0454 | 0472 | 0489 | 0507 | 3 | 6 | 9 | 12 | 15 |
| 3 | .0524 | 0542 | 0559 | 0577 | 0594 | 0612 | 0629 | 0647 | 0664 | 0682 | 3 | 6 | 9 | 12 | 15 |
| 4 | .0699 | 0717 | 0734 | 0752 | 0769 | 0787 | 0805 | 0822 | 0840 | 0857 | 3 | 6 | 9 | 12 | 15 |
| 5 | .0875 | 0892 | 0910 | 0928 | 0945 | 0963 | 0981 | 0998 | 1016 | 1033 | 3 | 6 | 9 | 12 | 15 |
| 6 | .1051 | 1069 | 1086 | 1104 | 1122 | 1139 | 1157 | 1175 | 1192 | 1210 | 3 | 6 | 9 | 12 | 15 |
| 7 | .1228 | 1246 | 1263 | 1281 | 1299 | 1317 | 1334 | 1352 | 1370 | 1388 | 3 | 6 | 9 | 12 | 15 |
| 8 | .1405 | 1423 | 1441 | 1459 | 1477 | 1495 | 1512 | 1530 | 1548 | 1566 | 3 | 6 | 9 | 12 | 15 |
| 9 | .1584 | 1602 | 1620 | 1638 | 1655 | 1673 | 1691 | 1709 | 1727 | 1745 | 3 | 6 | 9 | 12 | 15 |
| 10 | .1763 | 1781 | 1799 | 1817 | 1835 | 1853 | 1871 | 1890 | 1908 | 1926 | 3 | 6 | 9 | 12 | 15 |
| 11 | .1944 | 1962 | 1980 | 1998 | 2016 | 2035 | 2053 | 2071 | 2089 | 2107 | 3 | 6 | 9 | 12 | 15 |
| 12 | .2126 | 2144 | 2162 | 2180 | 2199 | 2217 | 2235 | 2254 | 2272 | 2290 | 3 | 6 | 9 | 12 | 15 |
| 13 | .2309 | 2327 | 2345 | 2364 | 2382 | 2401 | 2419 | 2438 | 2456 | 2475 | 3 | 6 | 9 | 12 | 15 |
| 14 | .2493 | 2512 | 2530 | 2549 | 2568 | 2586 | 2605 | 2623 | 2642 | 2661 | 3 | 6 | 9 | 12 | 16 |
| 15 | .2679 | 2698 | 2717 | 2736 | 2754 | 2773 | 2792 | 2811 | 2830 | 2849 | 3 | 6 | 9 | 13 | 16 |
| 16 | .2867 | 2886 | 2905 | 2924 | 2943 | 2962 | 2981 | 3000 | 3019 | 3038 | 3 | 6 | 9 | 13 | 16 |
| 17 | .3057 | 3076 | 3096 | 3115 | 3134 | 3153 | 3172 | 3191 | 3211 | 3230 | 3 | 6 | 10 | 13 | 16 |
| 18 | .3249 | 3269 | 3288 | 3307 | 3327 | 3346 | 3365 | 3385 | 3404 | 3424 | 3 | 6 | 10 | 13 | 16 |
| 19 | .3443 | 3463 | 3482 | 3502 | 3522 | 3541 | 3561 | 3581 | 3600 | 3620 | 3 | 6 | 10 | 13 | 17 |
| 20 | .3640 | 3659 | 3679 | 3699 | 3719 | 3739 | 3759 | 3779 | 3799 | 3819 | 3 | 7 | 10 | 13 | 17 |
| 21 | .3839 | 3859 | 3879 | 3899 | 3919 | 3939 | 3959 | 3979 | 4000 | 4020 | 3 | 7 | 10 | 13 | 17 |
| 22 | .4040 | 4061 | 4081 | 4101 | 4122 | 4142 | 4163 | 4183 | 4204 | 4224 | 3 | 7 | 10 | 14 | 17 |
| 23 | .4245 | 4265 | 4286 | 4307 | 4327 | 4348 | 4369 | 4390 | 4411 | 4431 | 3 | 7 | 10 | 14 | 17 |
| 24 | .4452 | 4473 | 4494 | 4515 | 4536 | 4557 | 4578 | 4599 | 4621 | 4642 | 4 | 7 | 10 | 14 | 18 |
| 25 | .4663 | 4684 | 4706 | 4727 | 4748 | 4770 | 4791 | 4813 | 4834 | 4856 | 4 | 7 | 11 | 14 | 18 |
| 26 | .4877 | 4899 | 4921 | 4942 | 4964 | 4986 | 5008 | 5029 | 5051 | 5073 | 4 | 7 | 11 | 15 | 18 |
| 27 | .5095 | 5117 | 5139 | 5161 | 5184 | 5206 | 5228 | 5250 | 5272 | 5295 | 4 | 7 | 11 | 15 | 18 |
| 28 | .5317 | 5340 | 5362 | 5384 | 5407 | 5430 | 5452 | 5475 | 5498 | 5520 | 4 | 8 | 11 | 15 | 19 |
| 29 | .5543 | 5566 | 5589 | 5612 | 5635 | 5658 | 5681 | 5704 | 5727 | 5750 | 4 | 8 | 12 | 15 | 19 |
| 30 | .5774 | 5797 | 5820 | 5844 | 5867 | 5890 | 5914 | 5938 | 5961 | 5985 | 4 | 8 | 12 | 16 | 20 |
| 31 | .6009 | 6032 | 6056 | 6080 | 6104 | 6128 | 6152 | 6176 | 6200 | 6224 | 4 | 8 | 12 | 16 | 20 |
| 32 | .6249 | 6273 | 6297 | 6322 | 6346 | 6371 | 6395 | 6420 | 6445 | 6469 | 4 | 8 | 12 | 16 | 20 |
| 33 | .6494 | 6519 | 6544 | 6569 | 6594 | 6619 | 6644 | 6669 | 6694 | 6720 | 4 | 8 | 13 | 17 | 21 |
| 34 | .6745 | 6771 | 6796 | 6822 | 6847 | 6873 | 6899 | 6924 | 6950 | 6976 | 4 | 9 | 13 | 17 | 21 |
| 35 | .7002 | 7028 | 7054 | 7080 | 7107 | 7133 | 7159 | 7186 | 7212 | 7239 | 4 | 9 | 13 | 18 | 22 |
| 36 | .7265 | 7292 | 7319 | 7346 | 7373 | 7400 | 7427 | 7454 | 7481 | 7508 | 5 | 9 | 14 | 18 | 23 |
| 37 | .7536 | 7563 | 7590 | 7618 | 7646 | 7673 | 7701 | 7729 | 7757 | 7785 | 5 | 9 | 14 | 18 | 23 |
| 38 | .7813 | 7841 | 7869 | 7898 | 7926 | 7954 | 7983 | 8012 | 8040 | 8069 | 5 | 10 | 14 | 19 | 24 |
| 39 | .8098 | 8127 | 8156 | 8185 | 8214 | 8243 | 8273 | 8302 | 8332 | 8361 | 5 | 10 | 15 | 20 | 24 |
| 40 | .8391 | 8421 | 8451 | 8481 | 8511 | 8541 | 8571 | 8601 | 8632 | 8662 | 5 | 10 | 15 | 20 | 25 |
| 41 | .8693 | 8724 | 8754 | 8785 | 8816 | 8847 | 8878 | 8910 | 8941 | 8972 | 5 | 10 | 16 | 21 | 26 |
| 42 | .9004 | 9036 | 9067 | 9099 | 9131 | 9163 | 9195 | 9228 | 9260 | 9293 | 5 | 11 | 16 | 21 | 27 |
| 43 | .9325 | 9358 | 9391 | 9424 | 9457 | 9490 | 9523 | 9556 | 9590 | 9623 | 6 | 11 | 17 | 22 | 28 |
| 44 | .9657 | 9691 | 9725 | 9759 | 9793 | 9827 | 9861 | 9896 | 9930 | 9965 | 6 | 11 | 17 | 23 | 29 |

TABLE VI.—NATURAL TANGENTS—*Continued*

| | 0′ | 6′ | 12′ | 18′ | 24′ | 30′ | 36′ | 42′ | 48′ | 54′ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45° | 1.0000 | 0035 | 0070 | 0105 | 0141 | 0176 | 0212 | 0247 | 0283 | 0319 | 6 | 12 | 18 | 24 | 30 |
| 46 | 1.0355 | 0392 | 0428 | 0464 | 0501 | 0538 | 0575 | 0612 | 0649 | 0686 | 6 | 12 | 18 | 25 | 31 |
| 47 | 1.0724 | 0761 | 0799 | 0837 | 0875 | 0913 | 0951 | 0990 | 1028 | 1067 | 6 | 13 | 19 | 25 | 32 |
| 48 | 1.1106 | 1145 | 1184 | 1224 | 1263 | 1303 | 1343 | 1383 | 1423 | 1463 | 7 | 13 | 20 | 26 | 33 |
| 49 | 1.1504 | 1544 | 1585 | 1626 | 1667 | 1708 | 1750 | 1792 | 1833 | 1875 | 7 | 14 | 21 | 28 | 34 |
| 50 | 1.1918 | 1960 | 2002 | 2045 | 2088 | 2131 | 2174 | 2218 | 2261 | 2305 | 7 | 14 | 22 | 29 | 36 |
| 51 | 1.2349 | 2393 | 2437 | 2482 | 2527 | 2572 | 2617 | 2662 | 2708 | 2753 | 8 | 15 | 23 | 30 | 38 |
| 52 | 1.2799 | 2846 | 2892 | 2938 | 2985 | 3032 | 3079 | 3127 | 3175 | 3222 | 8 | 16 | 23 | 31 | 39 |
| 53 | 1.3270 | 3319 | 3367 | 3416 | 3465 | 3514 | 3564 | 3613 | 3663 | 3713 | 8 | 17 | 25 | 33 | 41 |
| 54 | 1.3765 | 3814 | 3865 | 3916 | 3968 | 4019 | 4071 | 4124 | 4176 | 4229 | 9 | 17 | 26 | 34 | 43 |
| 55 | 1.4281 | 4335 | 4388 | 4442 | 4496 | 4550 | 4605 | 4659 | 4715 | 4770 | 9 | 18 | 27 | 36 | 45 |
| 56 | 1.4826 | 4882 | 4938 | 4994 | 5051 | 5108 | 5166 | 5224 | 5282 | 5340 | 10 | 19 | 29 | 38 | 48 |
| 57 | 1.5399 | 5458 | 5517 | 5577 | 5637 | 5697 | 5757 | 5818 | 5880 | 5941 | 10 | 20 | 30 | 40 | 50 |
| 58 | 1.6003 | 6066 | 6128 | 6191 | 6255 | 6319 | 6383 | 6440 | 6512 | 6577 | 11 | 21 | 32 | 43 | 53 |
| 59 | 1.6643 | 6709 | 6775 | 6842 | 6909 | 6977 | 7045 | 7113 | 7182 | 7251 | 11 | 23 | 34 | 45 | 56 |
| 60 | 1.7321 | 7391 | 7461 | 7532 | 7603 | 7675 | 7747 | 7820 | 7893 | 7966 | 12 | 24 | 36 | 48 | 60 |
| 61 | 1.8040 | 8115 | 8190 | 8265 | 8341 | 8418 | 8495 | 8572 | 8650 | 8728 | 13 | 26 | 38 | 51 | 64 |
| 62 | 1.8807 | 8887 | 8967 | 9047 | 9128 | 9210 | 9292 | 9375 | 9458 | 9542 | 14 | 27 | 41 | 55 | 68 |
| 63 | 1.9626 | 9711 | 9797 | 9883 | 9970 | 0057 | 0145 | 0233 | 0323 | 0413 | 15 | 29 | 44 | 58 | 73 |
| 64 | 2.0503 | 0594 | 0686 | 0778 | 0872 | 0965 | 1060 | 1155 | 1251 | 1348 | 16 | 31 | 47 | 63 | 78 |
| 65 | 2.1445 | 1543 | 1642 | 1742 | 1842 | 1943 | 2045 | 2148 | 2251 | 2355 | 17 | 34 | 51 | 68 | 85 |
| 66 | 2.2460 | 2566 | 2673 | 2781 | 2889 | 2998 | 3109 | 3220 | 3332 | 3445 | 18 | 37 | 55 | 74 | 92 |
| 67 | 2.3559 | 3673 | 3789 | 3906 | 4023 | 4142 | 4262 | 4383 | 4504 | 4627 | 20 | 40 | 60 | 79 | 99 |
| 68 | 2.4751 | 4876 | 5002 | 5129 | 5257 | 5386 | 5517 | 5649 | 5782 | 5916 | 22 | 43 | 65 | 87 | 108 |
| 69 | 2.6051 | 6187 | 6325 | 6464 | 6605 | 6746 | 6889 | 7034 | 7179 | 7326 | 24 | 47 | 71 | 95 | 118 |
| 70 | 2.7475 | 7625 | 7776 | 7929 | 8083 | 8239 | 8397 | 8556 | 8716 | 8878 | 26 | 52 | 78 | 104 | 130 |
| 71 | 2.9042 | 9208 | 9375 | 9544 | 9714 | 9887 | 0061 | 0237 | 0415 | 0595 | 29 | 58 | 87 | 115 | 144 |
| 72 | 3.0777 | 0961 | 1146 | 1334 | 1524 | 1716 | 1910 | 2106 | 2305 | 2506 | 32 | 64 | 96 | 129 | 161 |
| 73 | 3.2709 | 2914 | 3122 | 3332 | 3544 | 3759 | 3977 | 4197 | 4420 | 4646 | 36 | 72 | 108 | 144 | 180 |
| 74 | 3.4874 | 5105 | 5339 | 5576 | 5816 | 6059 | 6305 | 6554 | 6806 | 7062 | 41 | 82 | 122 | 162 | 203 |
| 75 | 3.7321 | 7583 | 7848 | 8118 | 8391 | 8667 | 8947 | 9232 | 9520 | 9812 | 46 | 94 | 139 | 186 | 232 |
| 76 | 4.0108 | 0408 | 0713 | 1022 | 1335 | 1653 | 1976 | 2303 | 2635 | 2972 | 53 | 107 | 160 | 214 | 267 |
| 77 | 4.3315 | 3662 | 4015 | 4374 | 4737 | 5107 | 5483 | 5864 | 6252 | 6646 | 62 | 124 | 186 | 248 | 310 |
| 78 | 4.7046 | 7453 | 7867 | 8288 | 8716 | 9152 | 9594 | 0045 | 0504 | 0970 | 73 | 146 | 219 | 292 | 365 |
| 79 | 5.1446 | 1929 | 2422 | 2924 | 3435 | 3955 | 4486 | 5026 | 5578 | 6140 | 87 | 175 | 262 | 350 | 437 |
| 80 | 5.6713 | 7297 | 7894 | 8502 | 9124 | 9758 | 0405 | 1066 | 1742 | 2432 | Difference columns cease to be useful owing to the rapidity with which the value of the tangent changes. | | | | |
| 81 | 6.3138 | 3859 | 4596 | 5350 | 6122 | 6912 | 7720 | 8548 | 9395 | 0264 | | | | | |
| 82 | 7.1154 | 2066 | 3002 | 3962 | 4947 | 5958 | 6996 | 8062 | 9158 | 0285 | | | | | |
| 83 | 8.1443 | 2636 | 3863 | 5126 | 6427 | 7769 | 9152 | 0579 | 2052 | 3572 | | | | | |
| 84 | 9.5144 | 9.677 | 9.845 | 10.02 | 10.20 | 10.39 | 10.58 | 10.78 | 10.99 | 11.20 | | | | | |
| 85 | 11.43 | 11.66 | 11.91 | 12.16 | 12.43 | 12.71 | 13.00 | 13.30 | 13.62 | 13.95 | | | | | |
| 86 | 14.30 | 14.67 | 15.06 | 15.46 | 15.89 | 16.35 | 16.83 | 17.34 | 17.89 | 18.46 | | | | | |
| 87 | 19.08 | 19.74 | 20.45 | 21.20 | 22.02 | 22.90 | 23.86 | 24.90 | 26.03 | 27.27 | | | | | |
| 88 | 28.64 | 30.14 | 31.82 | 33.69 | 35.80 | 38.19 | 40.92 | 44.07 | 47.74 | 52.08 | | | | | |
| 89 | 57.29 | 66.63 | 71.62 | 81.85 | 95.49 | 114.6 | 143.2 | 191.0 | 286.5 | 573.0 | | | | | |

## TABLE VII.—SATURATED WATER VAPOR

Adapted from Smithsonian Tables

Showing pressure *P* (in millimeters of mercury) and density *D* in gm./cc. of aqueous vapor saturated at temperature *t*; or showing boiling point *t* of water and density *D* of steam corresponding to an outside pressure *P*.

| *t* | *P* | *D* | *t* | *P* | *D* |
|---|---|---|---|---|---|
| −10 | 2.0 | $2.2 \times 10^{-6}$ | 80.0 | 355.1 | 293.8 |
| − 9 | 2.1 | 2.4 | 85.0 | 433.5 | 354.1 |
| − 8 | 2.3 | 2.6 | 90.0 | 525.8 | 424.1 |
| − 7 | 2.6 | 2.8 | 91.0 | 546.1 | 439.5 |
| − 6 | 2.8 | 3.0 | 92.0 | 567.1 | 455.2 |
| − 5 | 3.0 | 3.3 | 93.0 | 588.7 | 471.3 |
| − 4 | 3.3 | 3.5 | 94.0 | 611.0 | 487.8 |
| − 3 | 3.6 | 3.8 | 95.0 | 634.0 | 505 |
| − 2 | 3.9 | 4.1 | 96.0 | 657.7 | 523 |
| − 1 | 4.2 | 4.5 | 96.5 | 669.8 | |
| 0 | 4.6 | 4.9 | 97.0 | 682.1 | 541 |
| 1 | 4.9 | 5.2 | 97.5 | 694.5 | |
| 2 | 5.3 | 5.6 | 98.0 | 707.3 | 560 |
| 3 | 5.7 | 5.9 | 98.2 | 712.5 | |
| 4 | 6.1 | 6.4 | 98.4 | 717.6 | |
| 5 | 6.5 | 6.8 | 98.6 | 722.8 | |
| 6 | 7.0 | 7.3 | 98.8 | 728.0 | |
| 7 | 7.5 | 7.8 | 99.0 | 733.3 | 579 |
| 8 | 8.0 | 8.3 | 99.2 | 738.6 | |
| 9 | 8.6 | 8.8 | 99.4 | 743.9 | |
| 10 | 9.2 | 9.4 | 99.6 | 749.3 | |
| 11 | 9.8 | 10.0 | 99.8 | 754.7 | |
| 12 | 10.5 | 10.7 | 100.0 | 760.0 | 598 |
| 13 | 11.2 | 11.4 | 100.2 | 765.5 | |
| 14 | 12.0 | 12.1 | 100.4 | 770.9 | |
| 15 | 12.8 | 12.8 | 100.6 | 776.4 | |
| 16 | 13.6 | 13.6 | 100.8 | 781.9 | |
| 17 | 14.5 | 14.5 | 101 | 787.5 | 618 |
| 18 | 15.5 | 15.4 | 102 | 815.9 | 639 |
| 19 | 16.5 | 16.3 | 103 | 845.1 | 661 |
| 20 | 17.6 | 17.3 | 104 | 875.1 | 683 |
| 21 | 18.7 | 18.3 | 105 | 906.1 | 705 |
| 22 | 19.8 | 19.4 | 106 | 937.9 | 728 |
| 23 | 21.1 | 20.6 | 107 | 970.6 | 751 |
| 24 | 22.4 | 21.8 | 108 | 1004.3 | 776 |
| 25 | 23.8 | 23.0 | 109 | 1038.8 | 801 |
| 26 | 25.2 | 24.4 | 110 | 1074.5 | 827 |
| 27 | 26.8 | 25.8 | 112 | 1148.7 | 880 |
| 28 | 28.4 | 27.2 | 114 | 1227.1 | 936 |
| 29 | 30.1 | 28.8 | 116 | 1309.8 | 995 |
| 30 | 31.8 | 30.4 | 118 | 1397.0 | 1057 |
| 35 | 42.0 | 39.6 | 120 | 1489 | 1122 |
| 40 | 55.1 | 51.1 | 125 | 1740 | 1299 |
| 45 | 71.7 | 65.6 | 130 | 2026 | 1498 |
| 50 | 92.3 | 83.2 | 135 | 2348 | 1721 |
| 55 | 117.8 | 104.6 | 140 | 2710 | 1968 |
| 60 | 149.2 | 130.5 | 150 | 3569 | 2550 |
| 65 | 187.4 | 161.5 | 160 | 4633 | 3265 |
| 70 | 233.5 | 198.4 | 175 | 6689 | 4621 |
| 75 | 289.0 | 242.1 | 200 | 11650 | 7840 |

## TABLE VIII.—THE WET- AND DRY-BULB HYGROMETER

Adapted from Smithsonian Tables

Let the temperature of the atmosphere given by a dry-bulb thermometer be denoted by $t$°C., and let the reading of the wet-bulb thermometer be denoted by $(t - \Delta t)$. In the following table, corresponding to the various values of $\Delta t$ given in the top line, the pressure (in millimeters of mercury) of the aqueous vapor in the atmosphere at the temperature $t$°C. is given.

|  | Difference between the dry- and wet-bulb readings |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$°C. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 |
| −5 | 3.0 | 2.3 | 1.6 | 0.8 | 0.2 |  |  |  |  |  |  |  |  |
| −4 | 3.3 | 2.5 | 1.8 | 1.0 | 0.4 |  |  |  |  |  |  |  |  |
| −3 | 3.6 | 2.8 | 2.0 | 1.2 | 0.7 |  |  |  |  |  |  |  |  |
| −2 | 3.9 | 3.1 | 2.3 | 1.6 | 0.8 |  |  |  |  |  |  |  |  |
| −1 | 4.2 | 3.4 | 2.6 | 1.8 | 1.0 |  |  |  |  |  |  |  |  |
| 0 | 4.6 | 3.7 | 2.9 | 2.1 | 1.3 |  |  |  |  |  |  |  |  |
| 1 | 4.9 | 4.1 | 3.2 | 2.4 | 1.6 |  |  |  |  |  |  |  |  |
| 2 | 5.3 | 4.4 | 3.6 | 2.7 | 1.9 | 1.1 | 0.3 |  |  |  |  |  |  |
| 3 | 5.7 | 4.8 | 3.9 | 3.1 | 2.2 | 1.4 | 0.6 |  |  |  |  |  |  |
| 4 | 6.1 | 5.2 | 4.3 | 3.4 | 2.6 | 1.8 | 0.9 |  |  |  |  |  |  |
| 5 | 6.5 | 5.6 | 4.7 | 3.8 | 2.9 | 2.1 | 1.2 |  |  |  |  |  |  |
| 6 | 7.0 | 6.0 | 5.1 | 4.2 | 3.3 | 2.4 | 1.6 |  |  |  |  |  |  |
| 7 | 7.5 | 6.5 | 5.5 | 4.6 | 3.7 | 2.8 | 1.9 | 1.1 | 0.2 |  |  |  |  |
| 8 | 8.0 | 7.0 | 6.0 | 5.0 | 4.1 | 3.2 | 2.3 | 1.4 | 0.6 |  |  |  |  |
| 9 | 8.6 | 7.5 | 6.5 | 5.5 | 4.5 | 3.6 | 2.7 | 1.8 | 0.9 |  |  |  |  |
| 10 | 9.2 | 8.1 | 7.0 | 6.0 | 5.0 | 4.0 | 3.1 | 2.2 | 1.3 |  |  |  |  |
| 11 | 9.8 | 8.7 | 7.6 | 6.5 | 5.5 | 4.5 | 3.5 | 2.6 | 1.7 |  |  |  |  |
| 12 | 10.5 | 9.3 | 8.2 | 7.1 | 6.0 | 5.0 | 4.0 | 3.0 | 2.1 | 1.2 | 0.3 |  |  |
| 13 | 11.2 | 10.0 | 8.8 | 7.6 | 6.6 | 5.5 | 4.5 | 3.5 | 2.5 | 1.6 | 0.6 |  |  |
| 14 | 12.0 | 10.8 | 9.5 | 8.4 | 7.2 | 6.2 | 5.0 | 4.0 | 3.0 | 2.0 | 1.1 |  |  |
| 15 | 12.8 | 11.5 | 10.2 | 9.1 | 7.9 | 6.7 | 5.5 | 4.5 | 3.5 | 2.5 | 1.5 |  |  |
| 16 | 13.6 | 12.3 | 11.0 | 9.8 | 8.5 | 7.3 | 6.2 | 5.1 | 4.0 | 3.0 | 2.0 | 0.1 |  |
| 17 | 14.5 | 13.1 | 11.8 | 10.5 | 9.2 | 8.1 | 6.8 | 5.7 | 4.6 | 3.6 | 2.5 | 0.5 |  |
| 18 | 15.5 | 14.0 | 12.6 | 11.3 | 10.0 | 8.7 | 7.5 | 6.4 | 5.2 | 4.1 | 3.0 | 1.0 |  |
| 19 | 16.5 | 15.0 | 13.5 | 12.1 | 10.8 | 9.4 | 8.2 | 6.9 | 5.8 | 4.6 | 3.5 | 1.4 |  |
| 20 | 17.6 | 16.1 | 14.5 | 13.0 | 11.6 | 10.3 | 8.9 | 7.6 | 6.4 | 5.2 | 4.1 | 2.0 |  |
| 21 | 18.7 | 17.1 | 15.5 | 13.9 | 12.5 | 11.1 | 9.7 | 8.5 | 7.2 | 6.0 | 4.8 | 2.5 | 0.4 |
| 22 | 19.8 | 18.1 | 16.5 | 14.9 | 13.4 | 12.0 | 10.6 | 9.2 | 7.9 | 6.6 | 5.4 | 3.1 | 1.0 |
| 23 | 21.1 | 19.3 | 17.6 | 16.0 | 14.4 | 12.9 | 11.5 | 10.1 | 8.7 | 7.4 | 6.1 | 3.8 | 1.5 |
| 24 | 22.4 | 20.6 | 18.8 | 17.2 | 15.5 | 14.0 | 12.4 | 11.0 | 9.5 | 8.2 | 6.9 | 4.4 | 2.1 |
| 25 | 23.8 | 21.9 | 20.1 | 18.3 | 16.6 | 15.0 | 13.4 | 11.9 | 10.4 | 9.1 | 7.7 | 5.1 | 2.7 |
| 26 | 25.2 | 23.3 | 21.4 | 19.6 | 17.8 | 16.1 | 14.5 | 13.0 | 11.4 | 9.9 | 8.5 | 5.9 | 3.4 |
| 27 | 26.8 | 24.8 | 22.8 | 21.0 | 19.0 | 17.3 | 15.6 | 14.0 | 12.4 | 10.9 | 9.4 | 6.7 | 4.1 |
| 28 | 28.4 | 26.3 | 24.2 | 22.2 | 20.3 | 18.5 | 16.8 | 15.1 | 13.4 | 11.9 | 10.4 | 7.5 | 4.9 |
| 29 | 30.1 | 27.9 | 25.7 | 23.7 | 21.7 | 19.8 | 18.0 | 16.3 | 14.6 | 13.0 | 11.4 | 8.4 | 5.6 |
| 30 | 31.9 | 29.6 | 27.3 | 25.3 | 23.2 | 21.2 | 19.3 | 17.5 | 15.7 | 14.0 | 12.4 | 9.3 | 6.5 |

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